Wavelet Analysis for Noisy Time Series

February 2020

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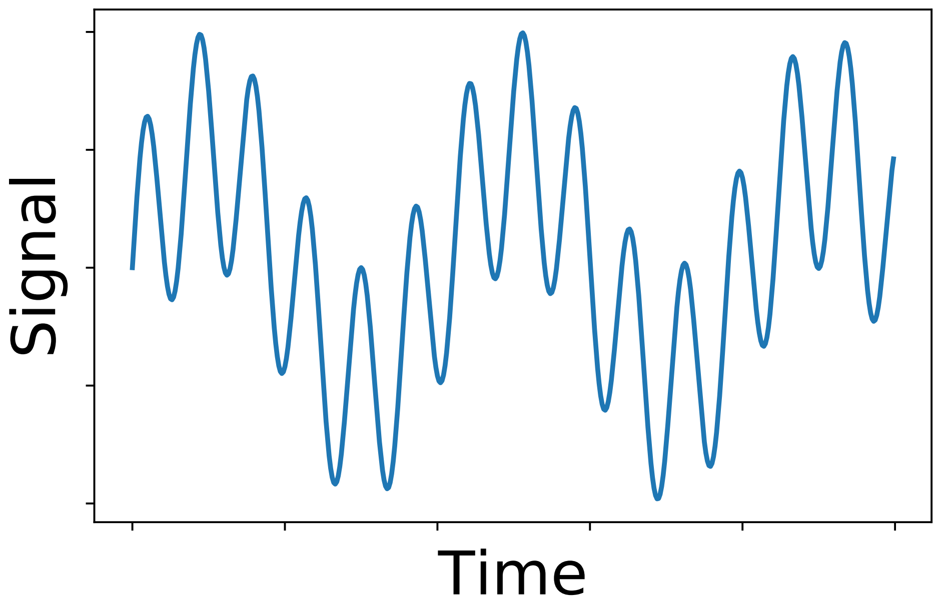
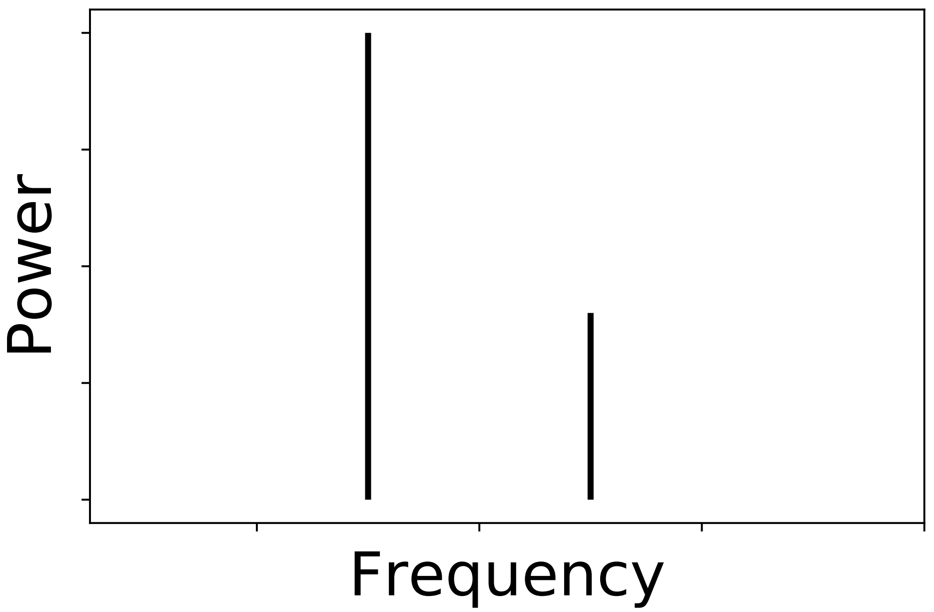
Recap: Frequency Analysis with Fourier

Time domain



$$\mathcal{F}: t \to \omega$$

Frequency domain



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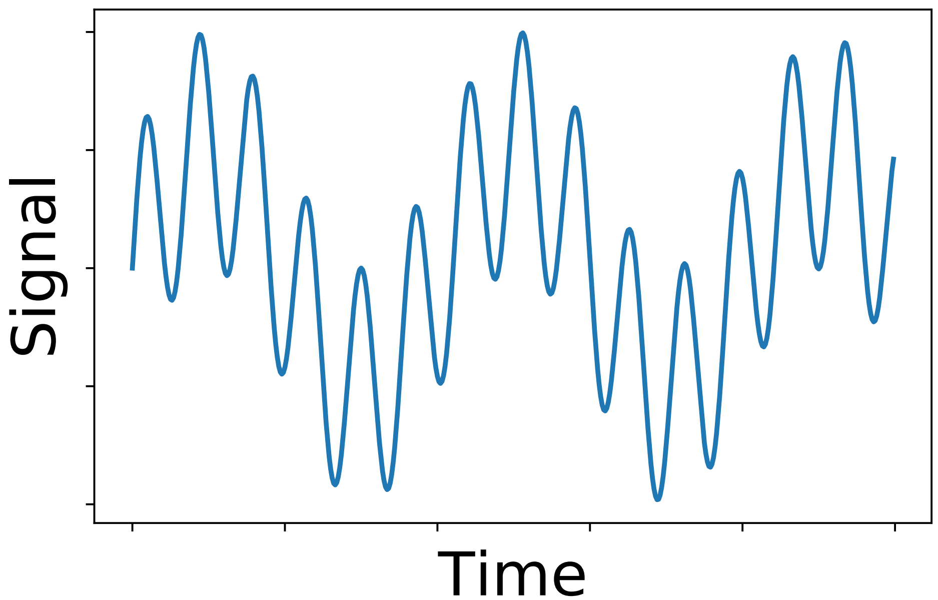

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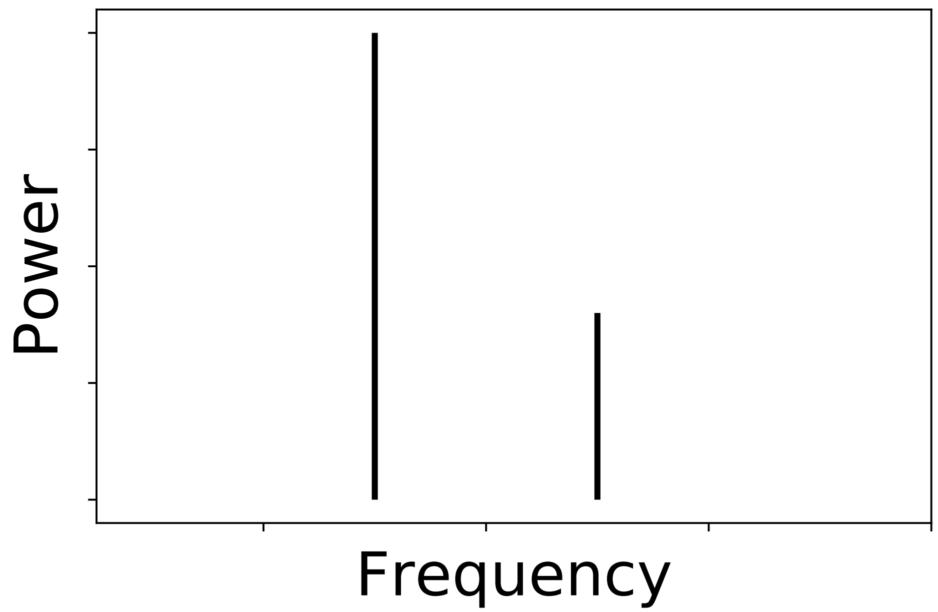
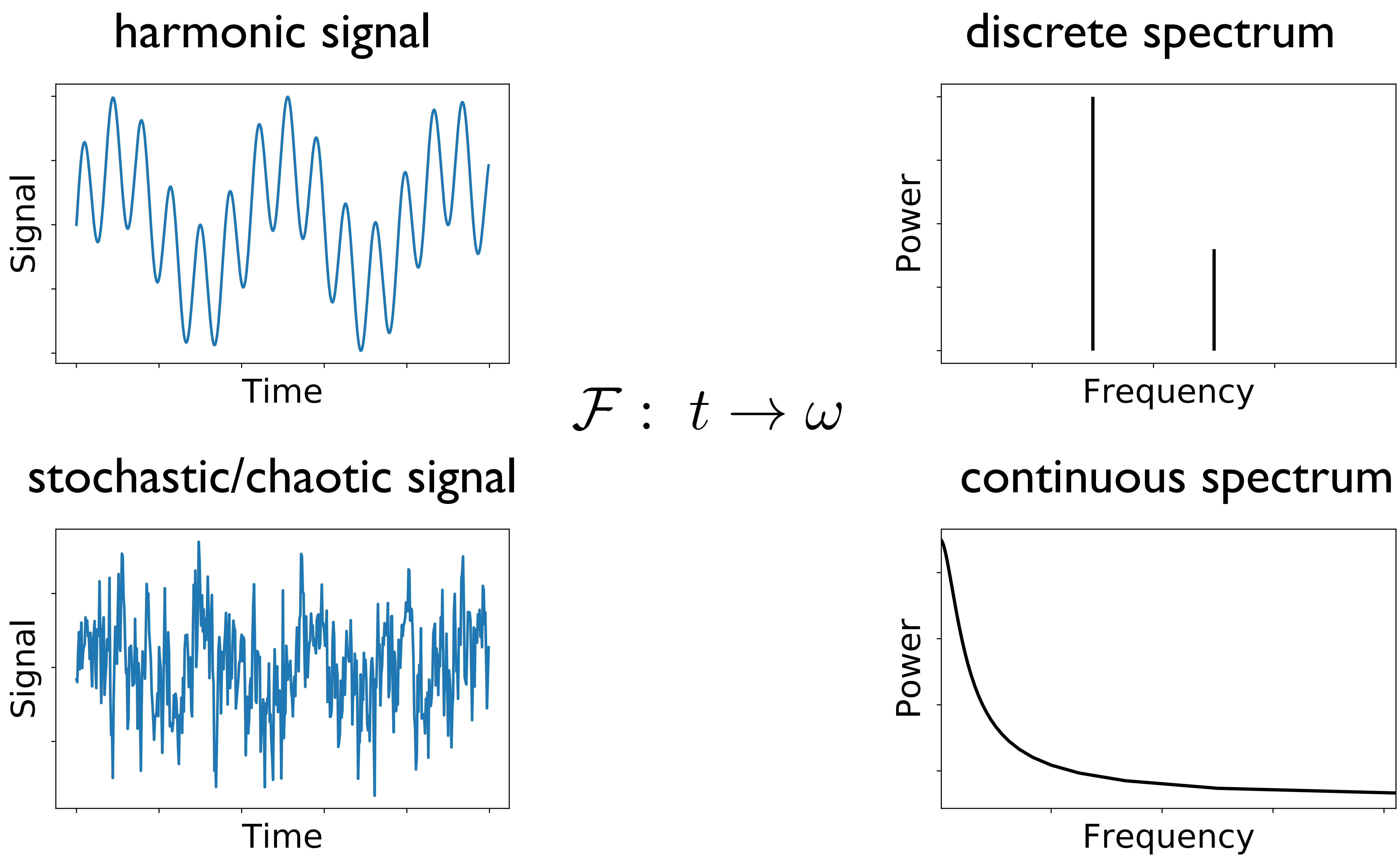
Frequency domain



Both representations contain
the same energy/information
(e.g. jpeg compression)

Discrete and Continuous Spectra

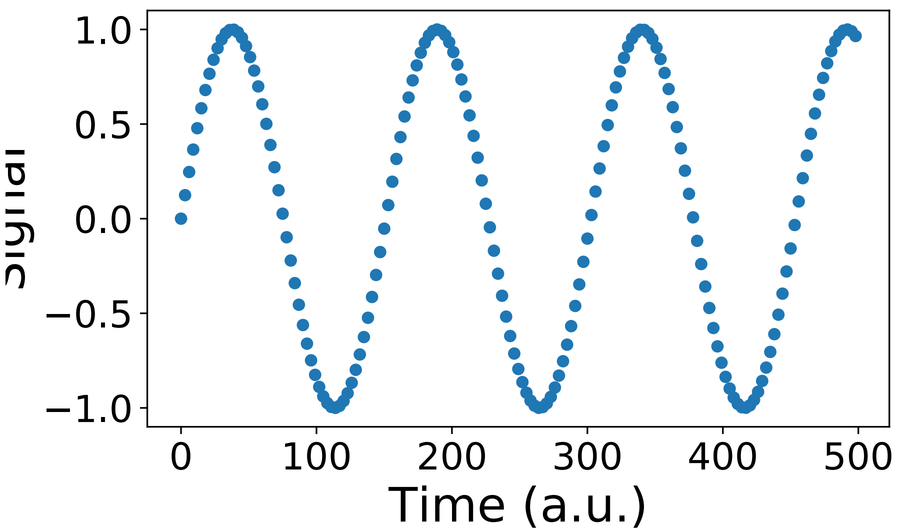


$$\mathcal{F} : t \rightarrow \omega$$

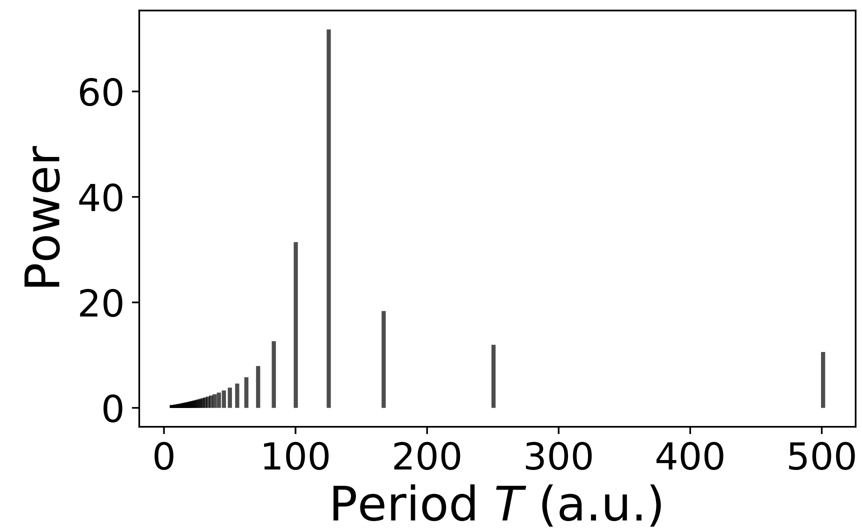
Most "real world" signals have both components!

Fourier Limitations

Short signal

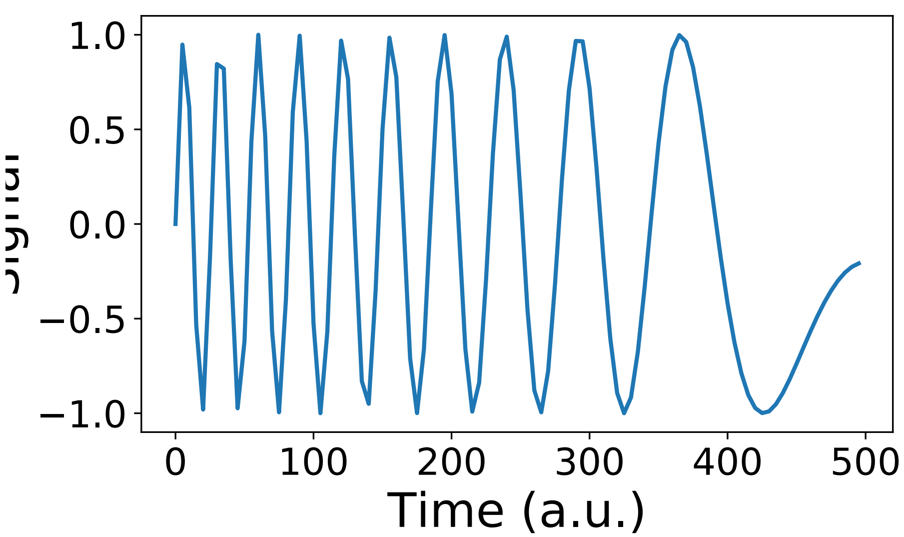


Poor spectral resolution

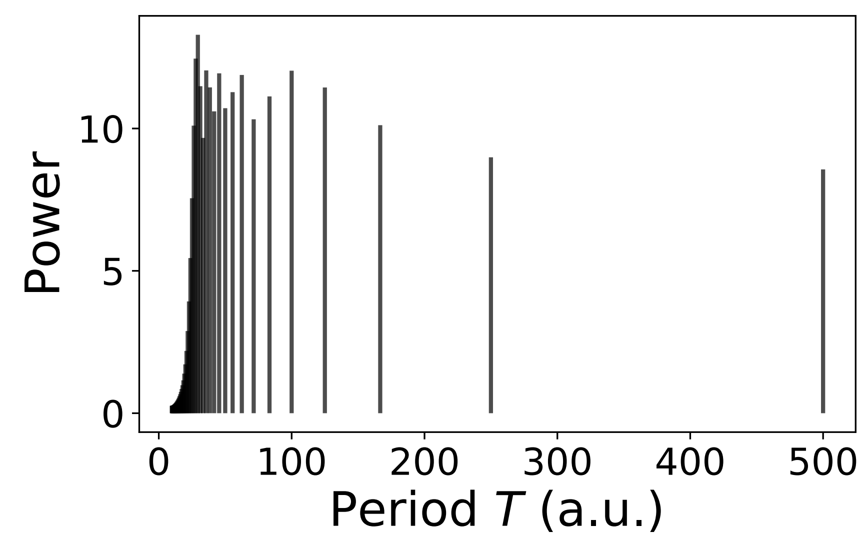


$$\mathcal{F}: \; t \rightarrow \omega$$

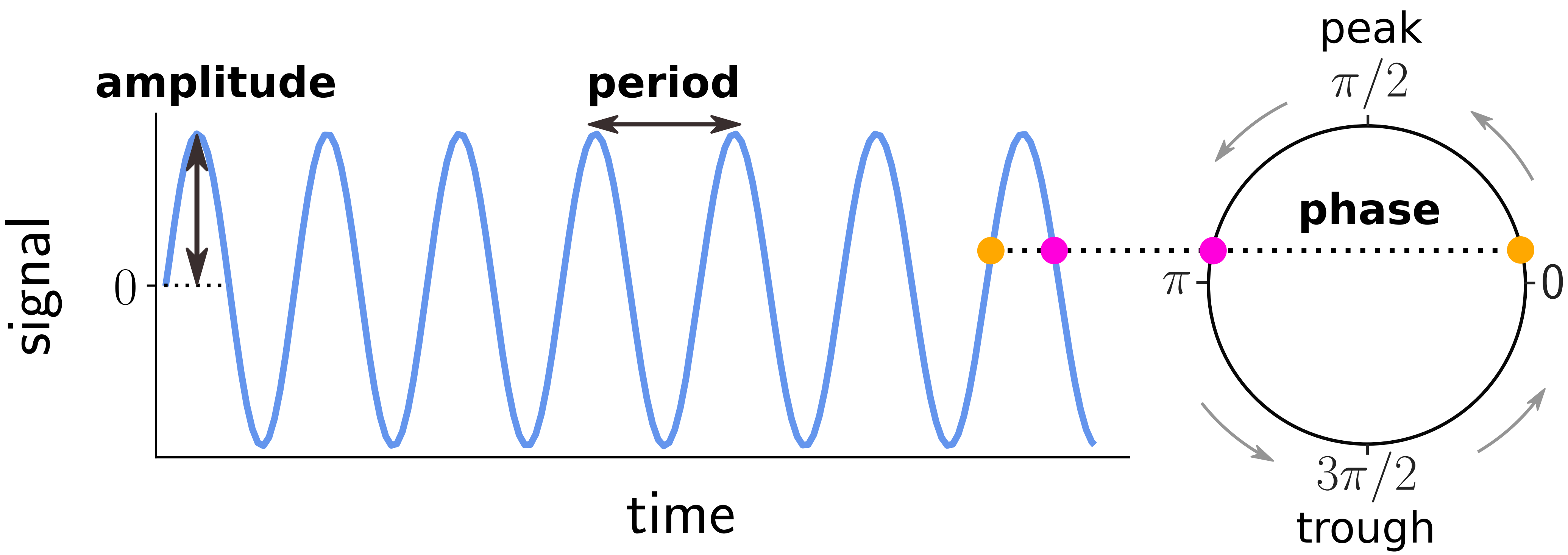
Non-stationary signal



No time-resolution



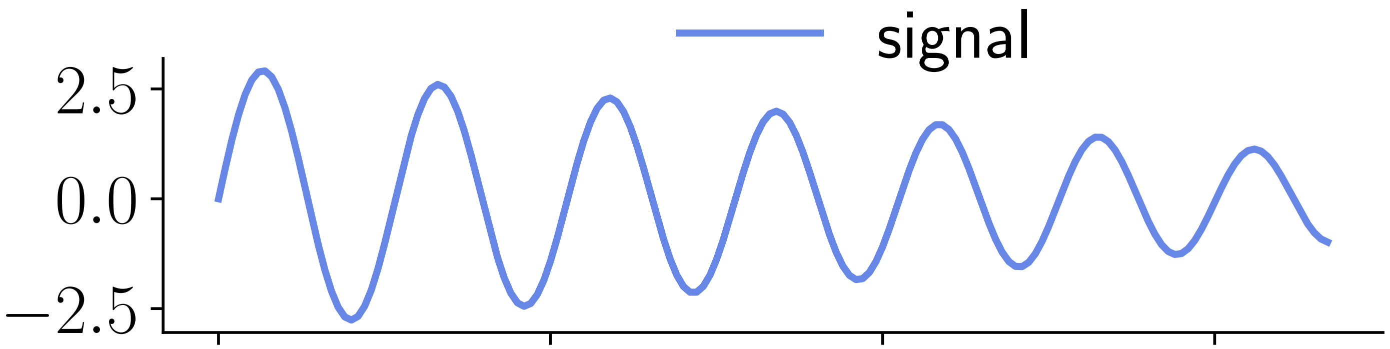
Problem Setting



- Amplitude and period potentially time-dependent!
- Uniquely characterize an **analytic signal**

$$A(t)\ cos[\phi(t)],\ \omega(t) = \frac{d\phi}{dt},\ \omega = \frac{2\pi}{T}$$

Problem Setting



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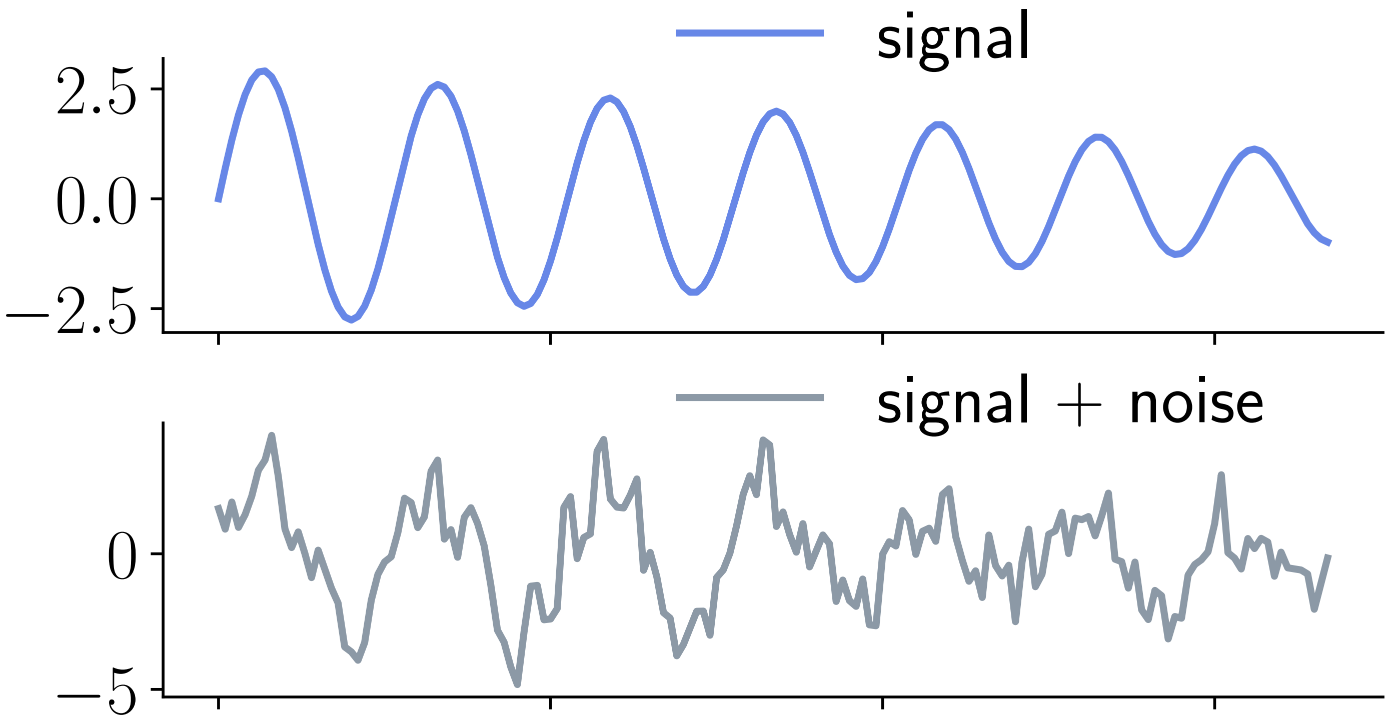

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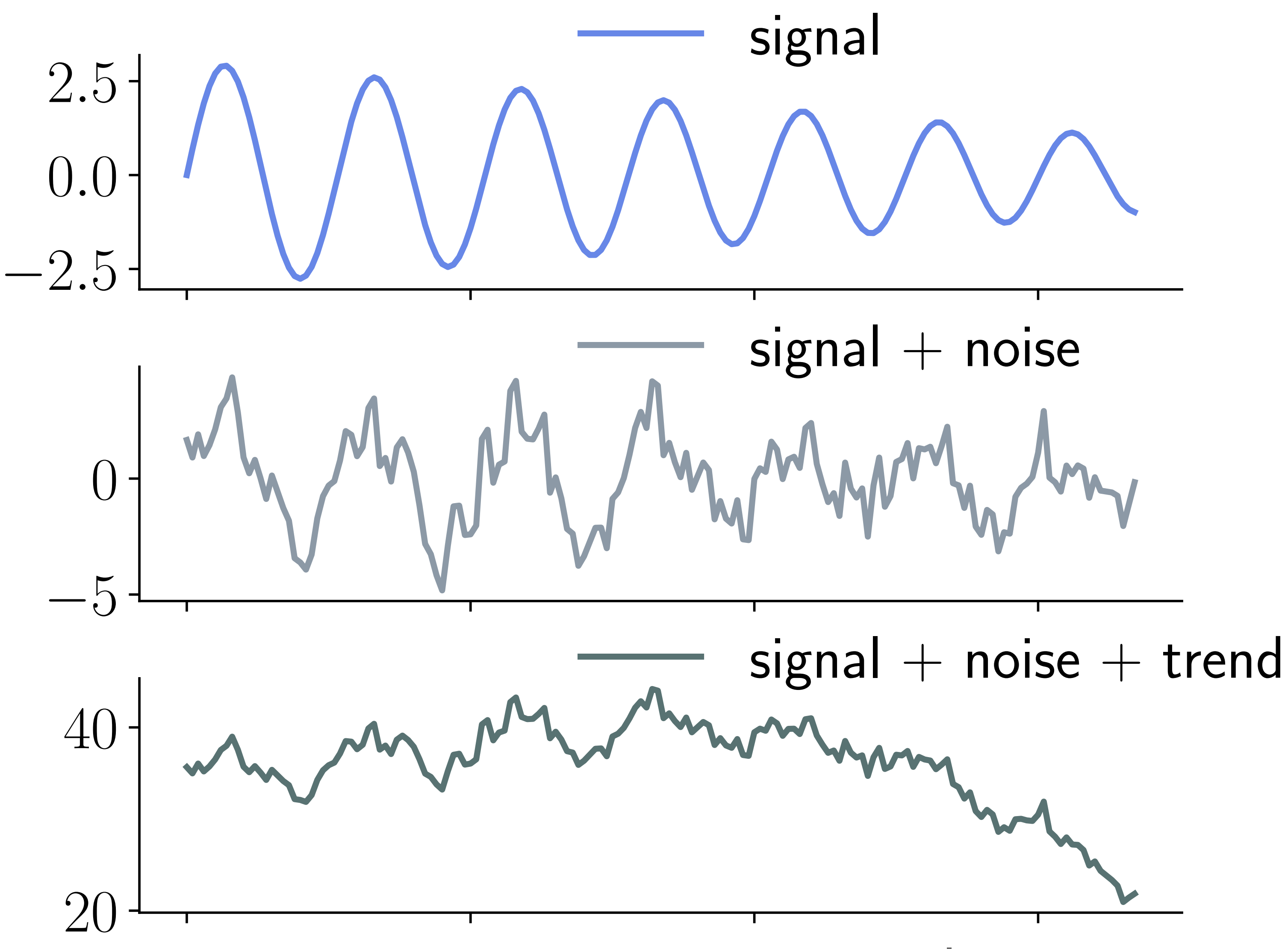

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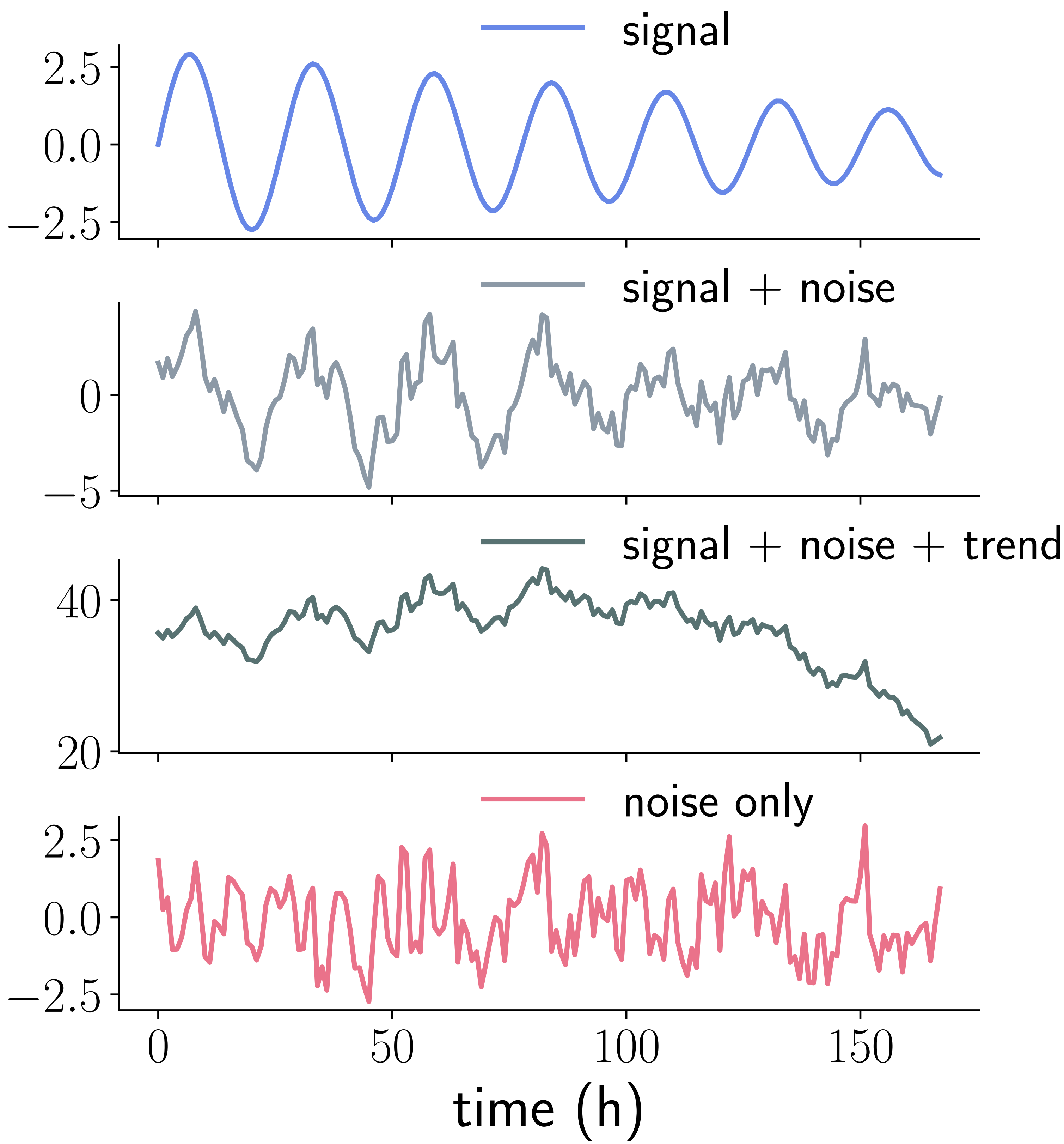
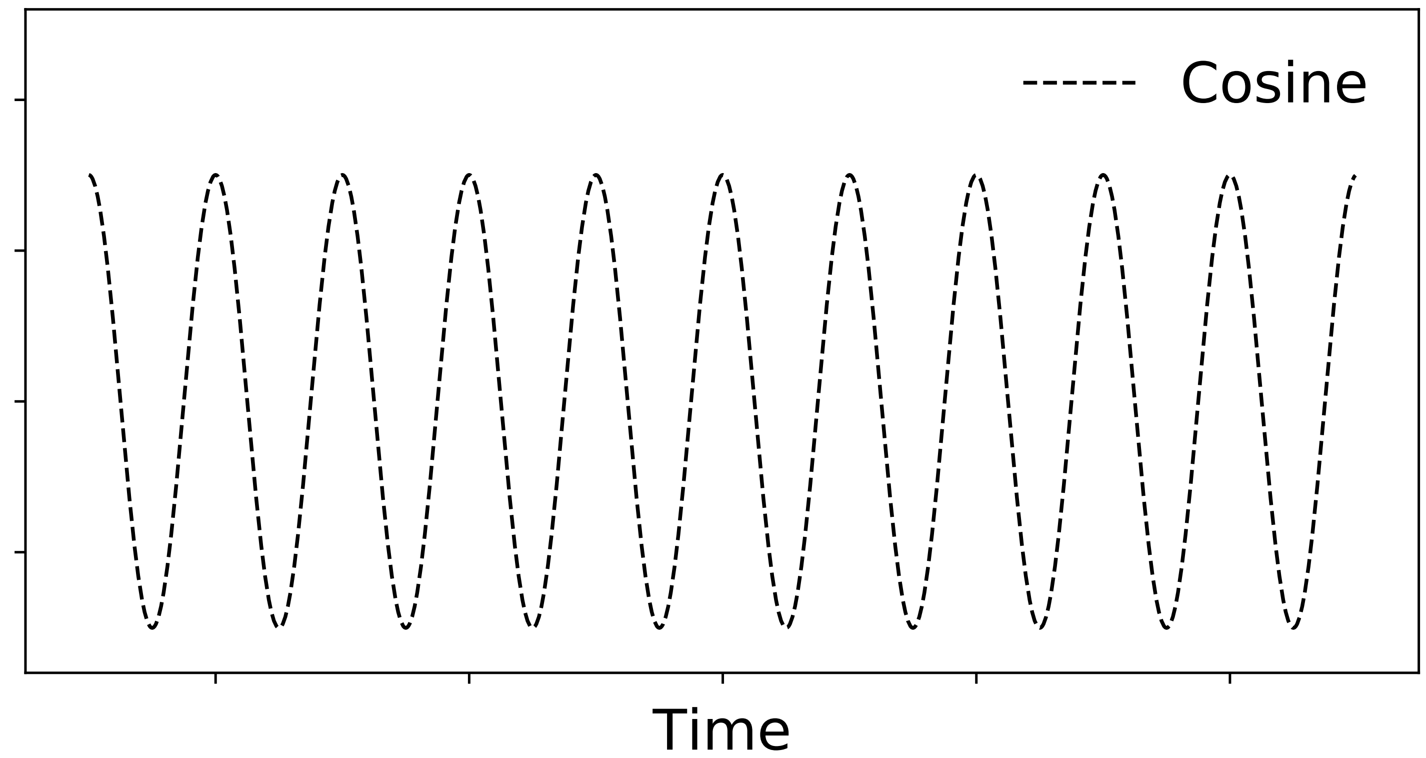


The Task:

- bias free estimation of period, phase and amplitude
- no spurious results

What are Wavelets?

- Fourier modes have no time localization



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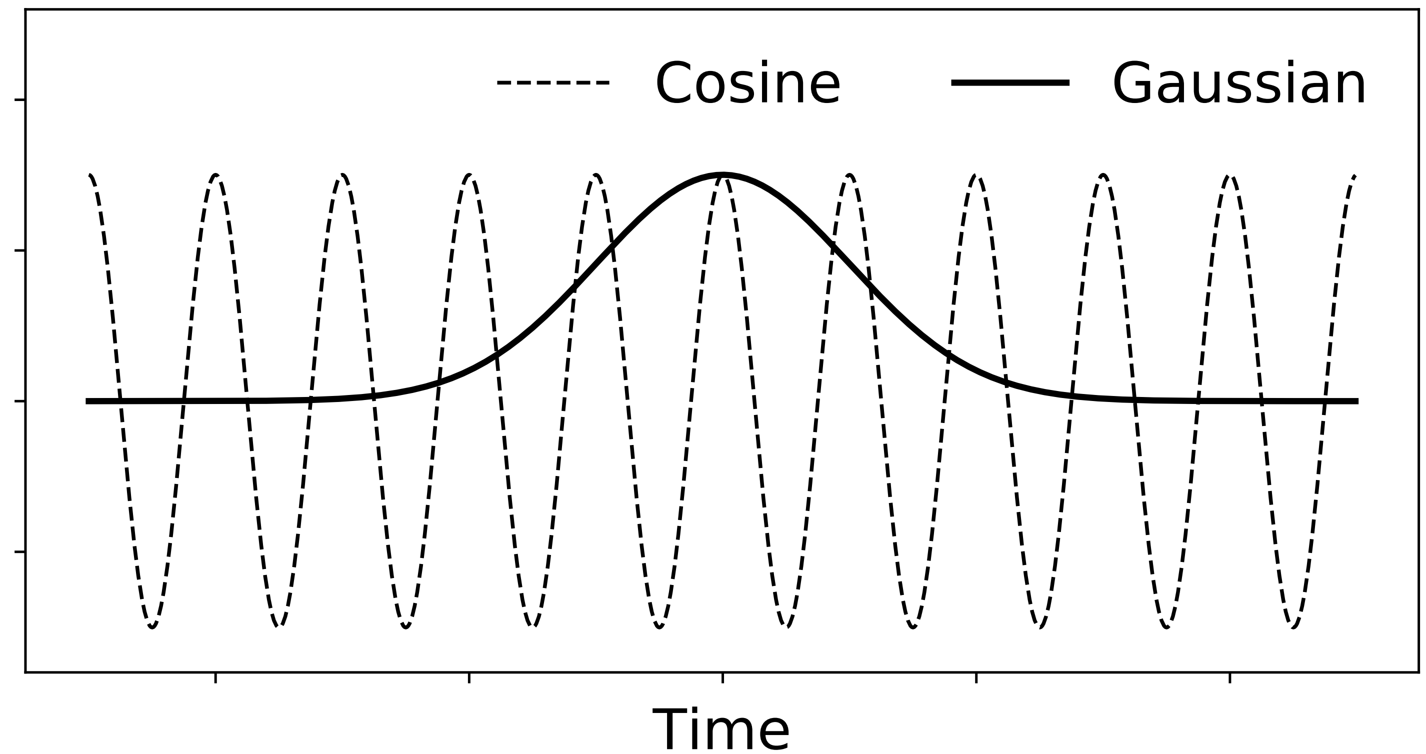

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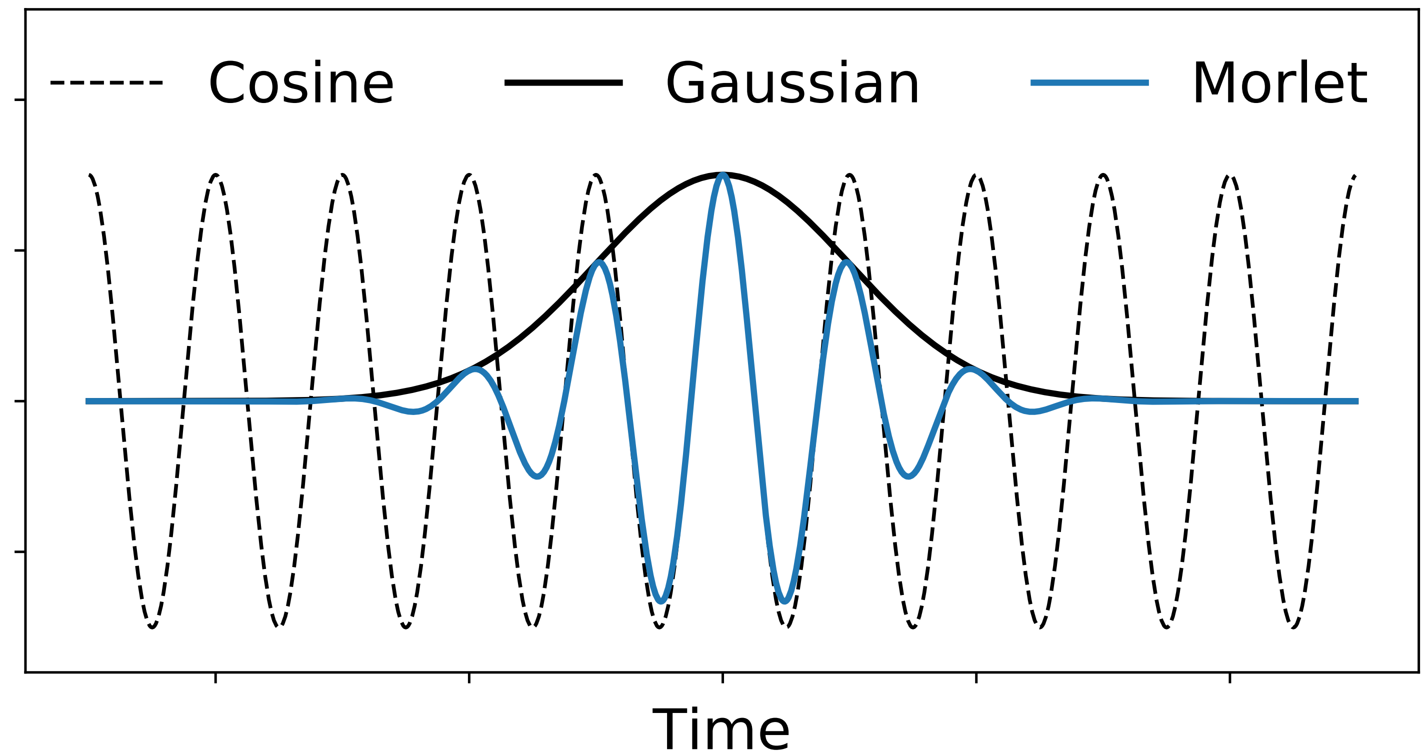

Morlet Mother Wavelet: $$\psi(t) = \pi^{1/4}\, e^{i\omega_0 t}\, e^{-\frac{1}{2}t^2}$$

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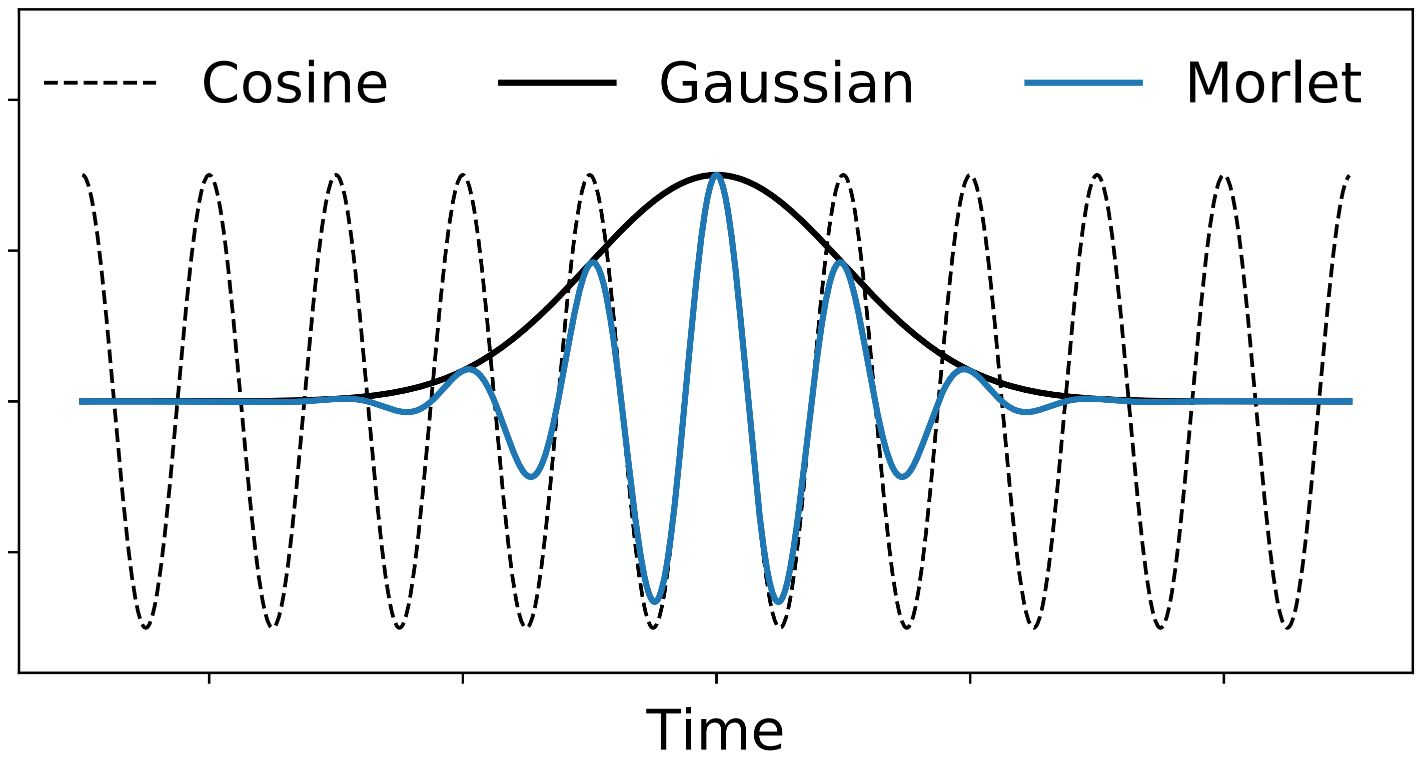

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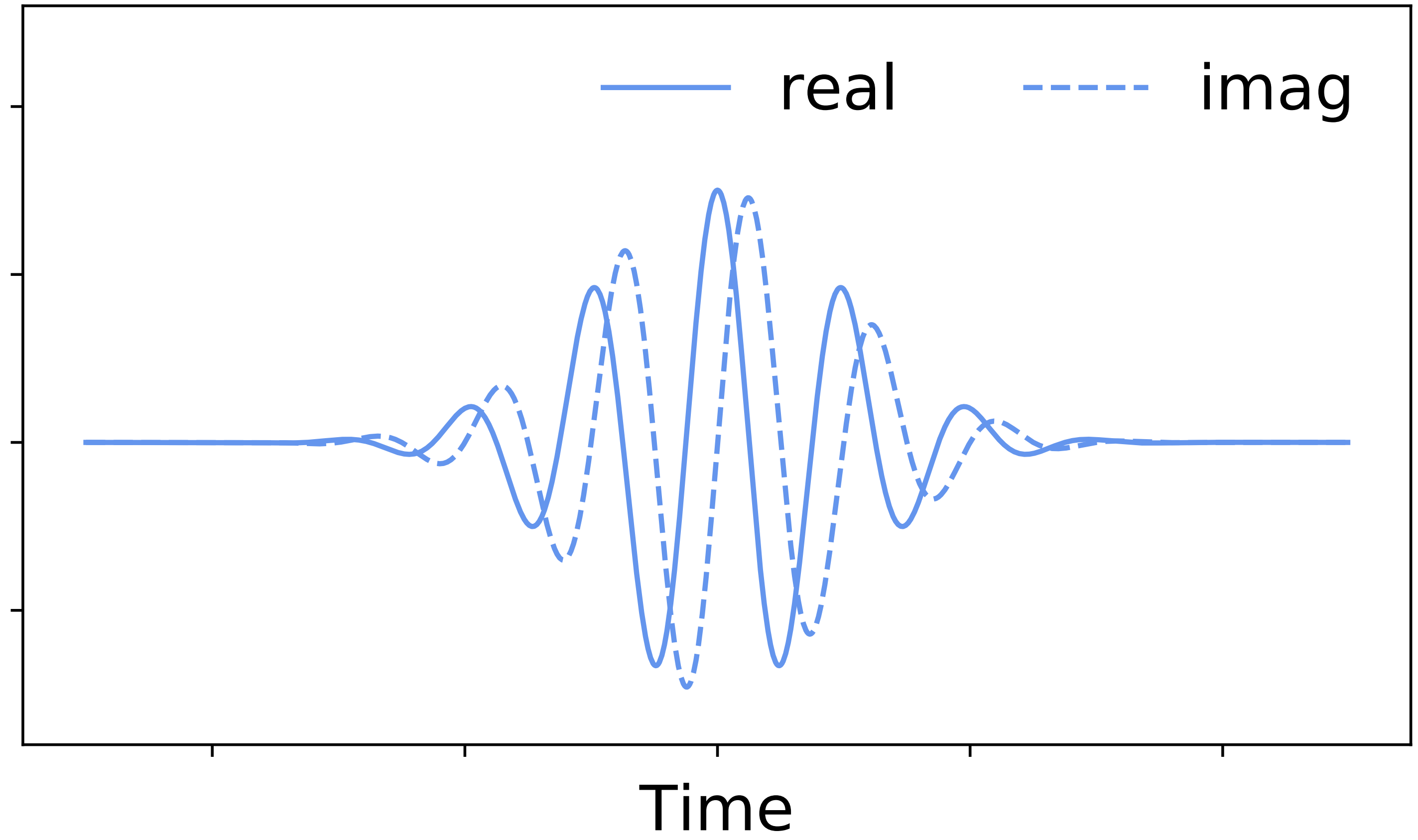
Complex Function!



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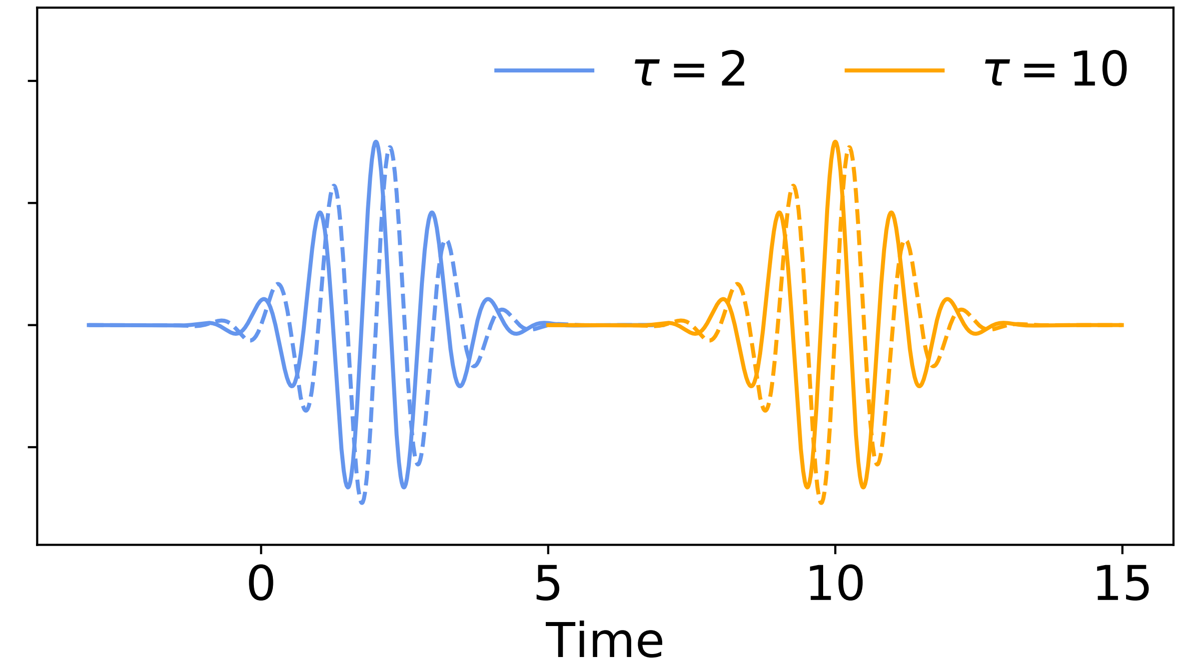
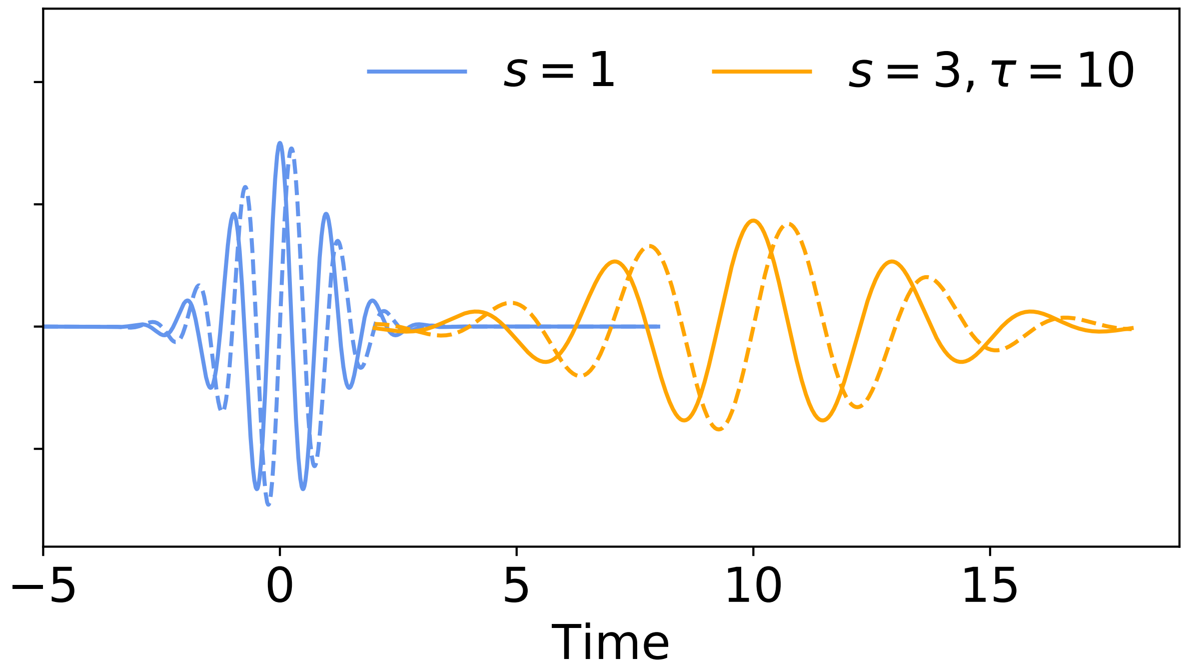
Translation and Dilation: We have a Family!

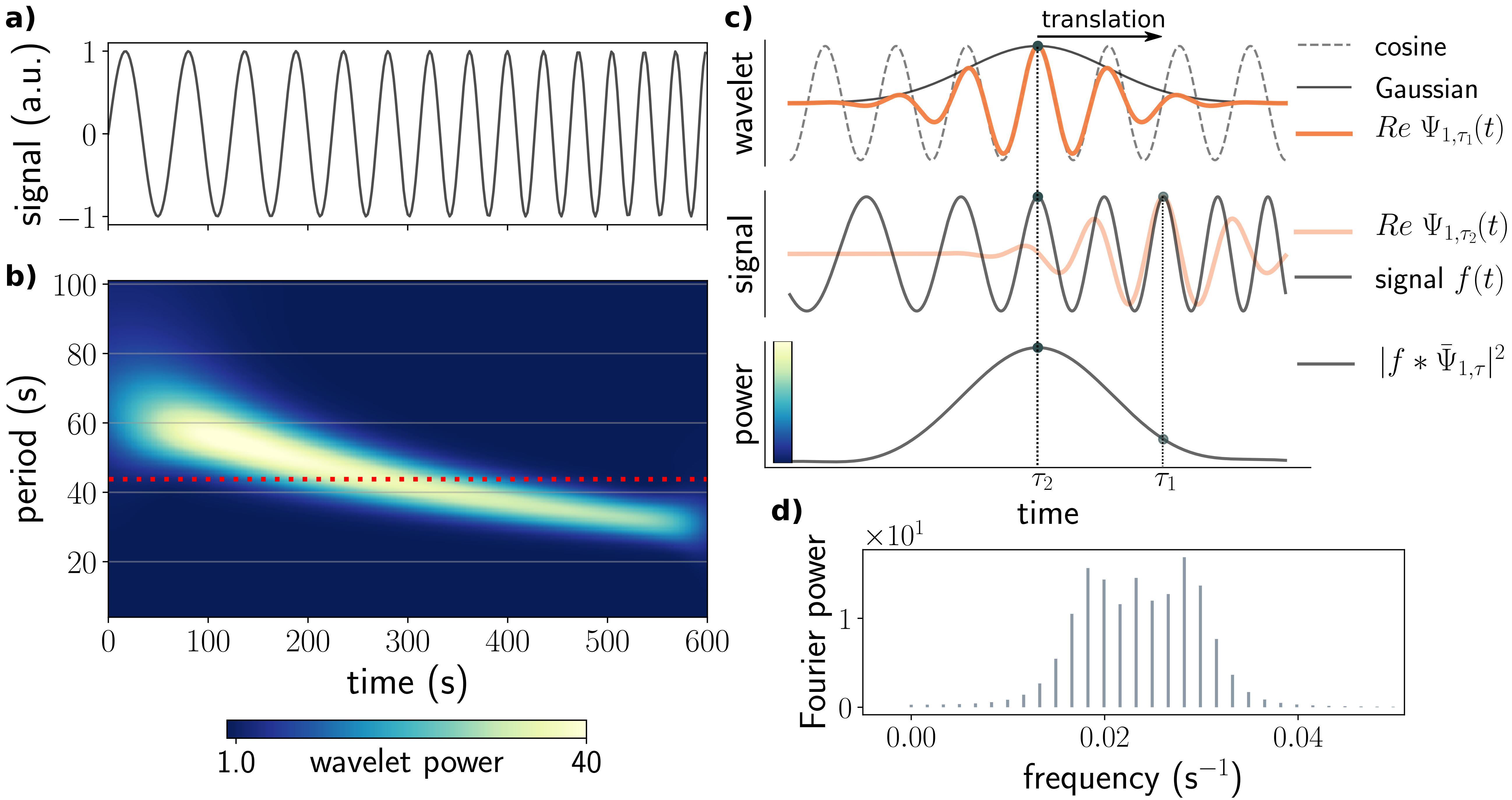
Shift in time: Translation



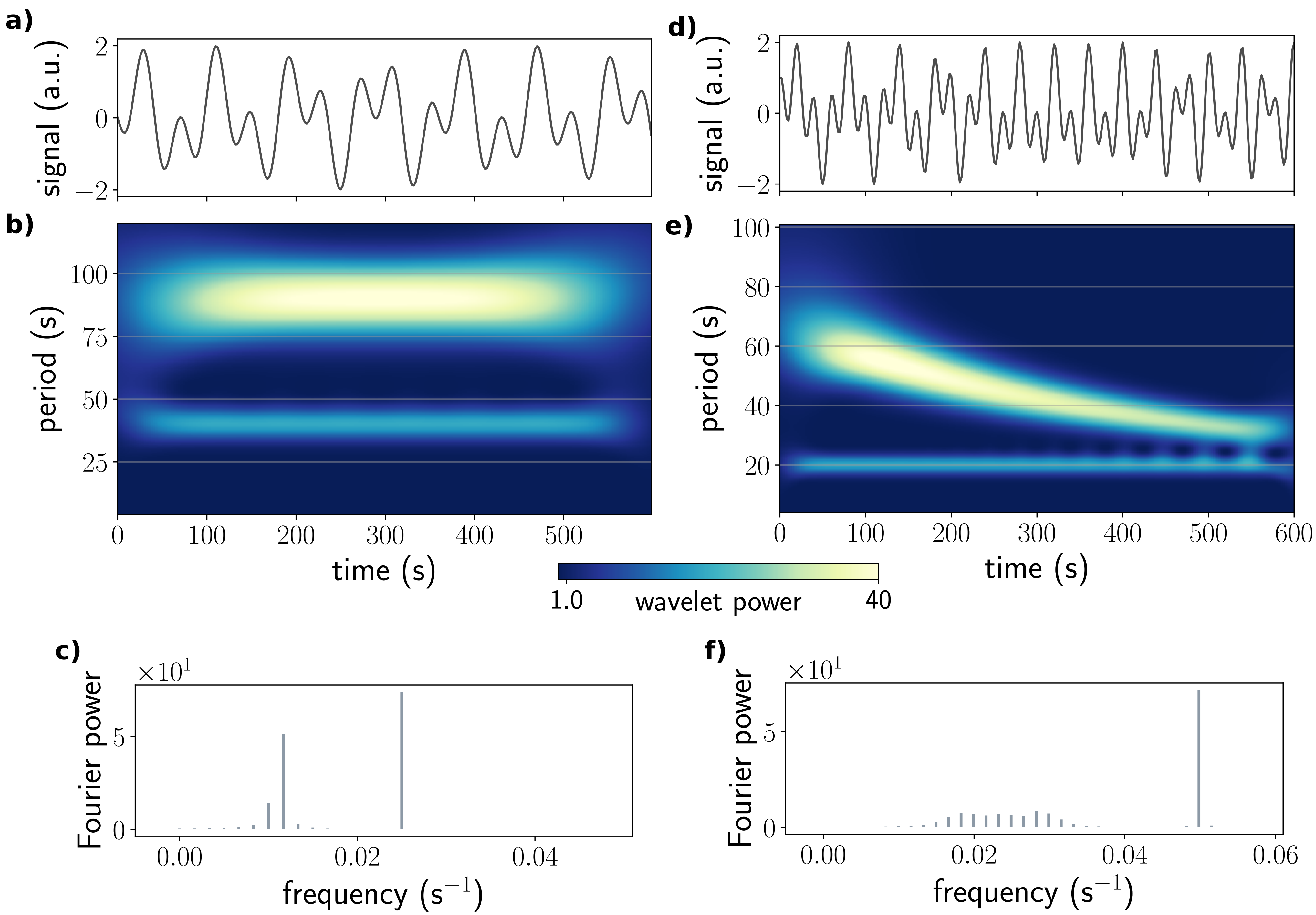
Change of wavelength: Dilation



Wavelet Analysis and Spectrum

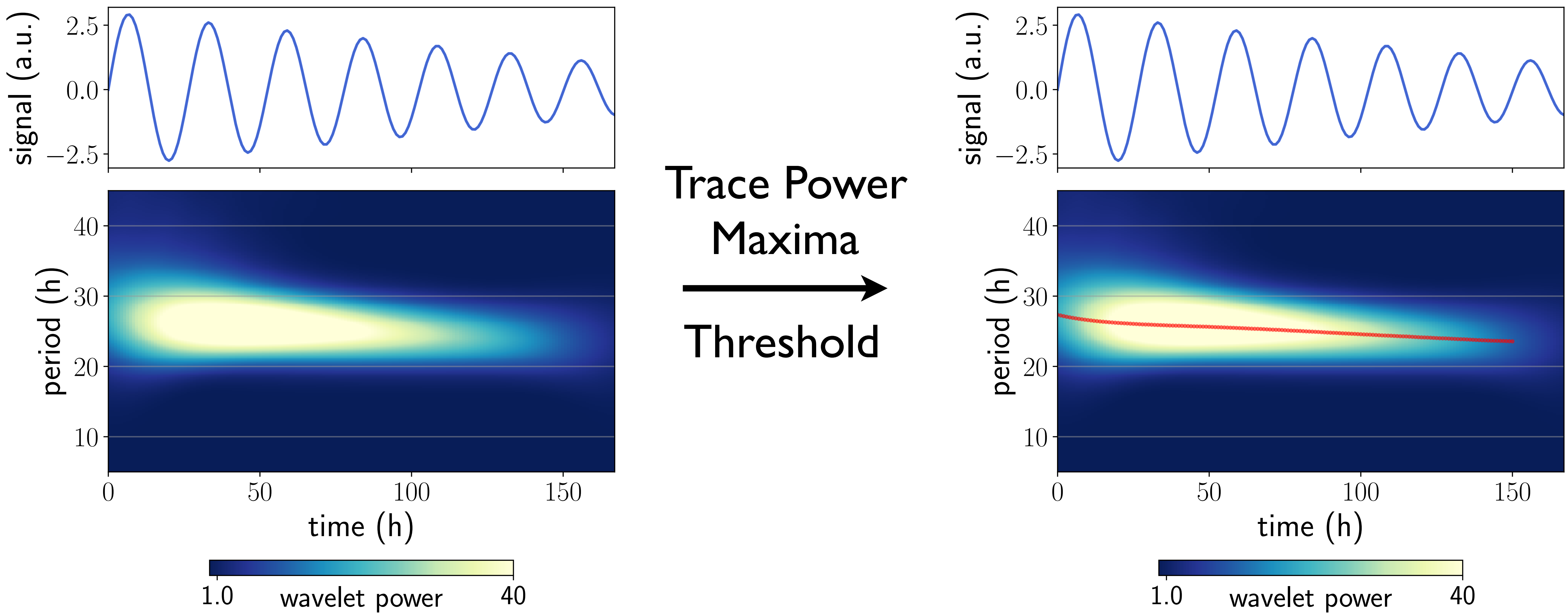


Asymptotic Spectrum



Time averaged Wavelet Spectrum is (optimal) estimate for the Fourier Spectrum!

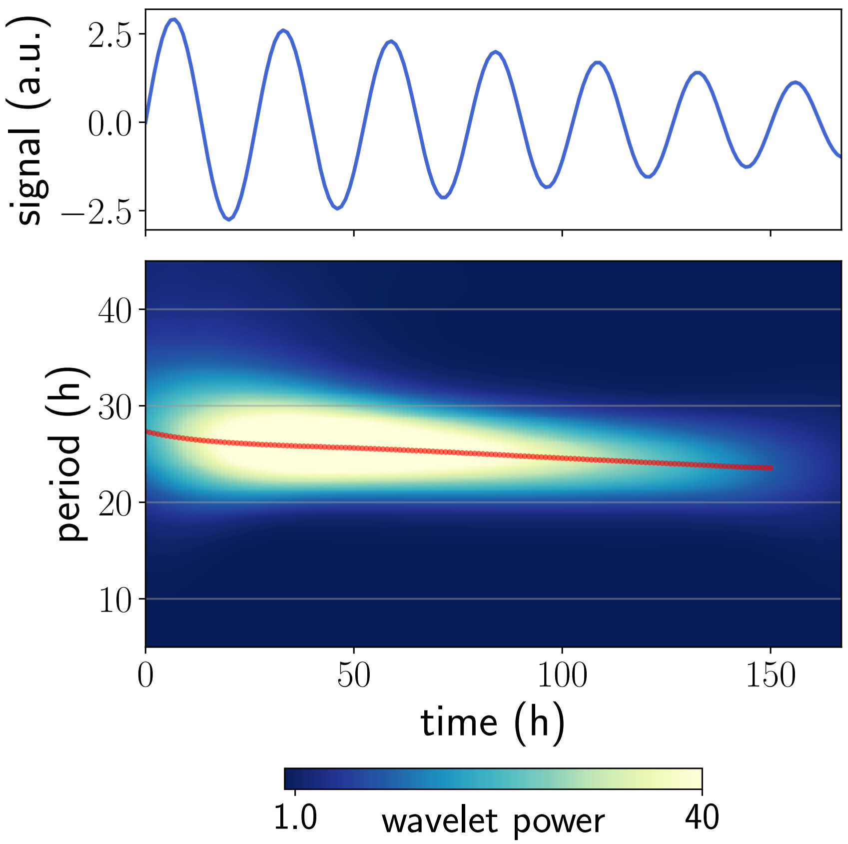
Ridge Extraction



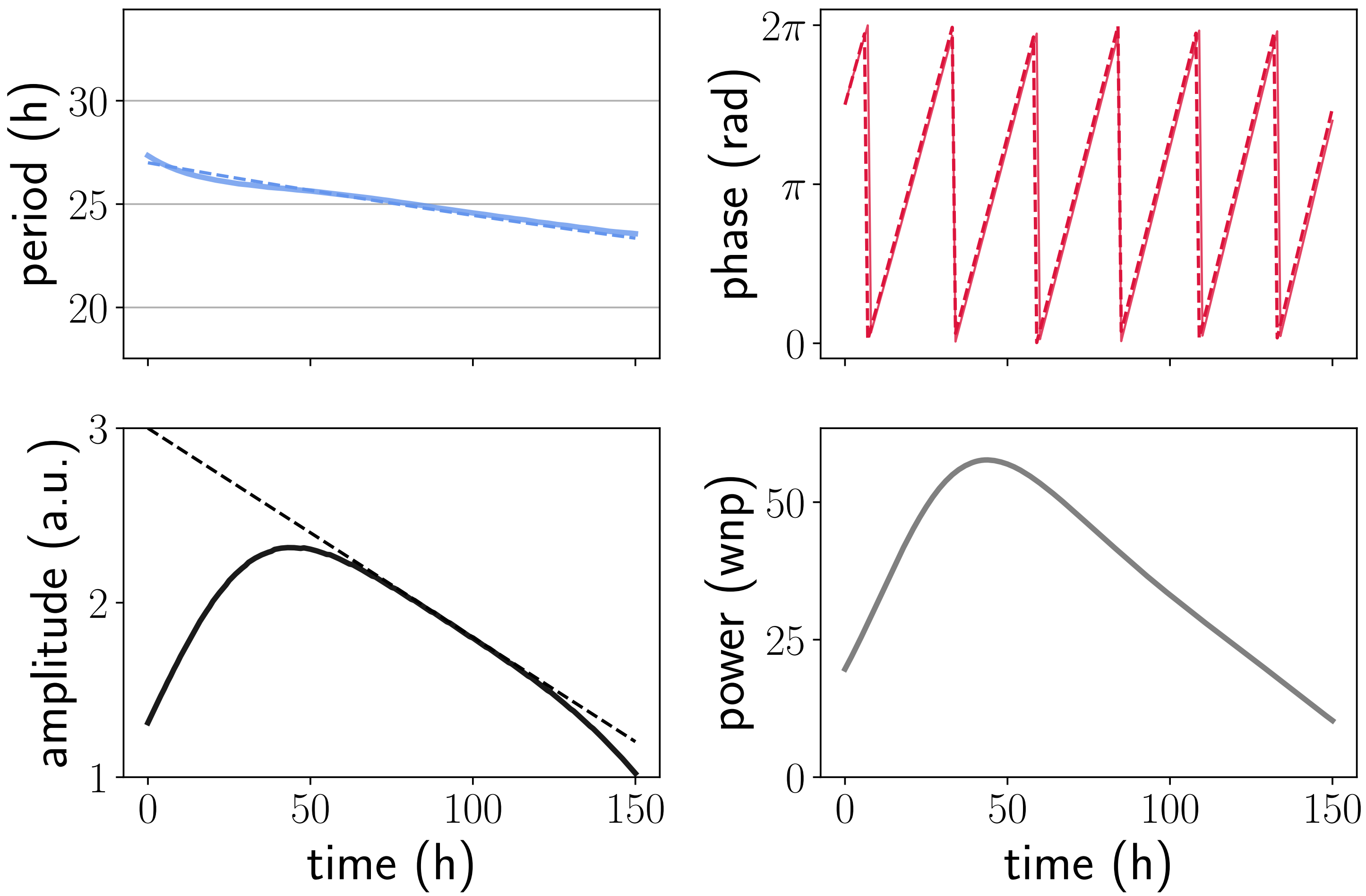
Wavelet power > 3 corresponds to 95% confidence interval for white noise

"A Practical Guide to Wavelet Analysis", Torrence and Compo 1997

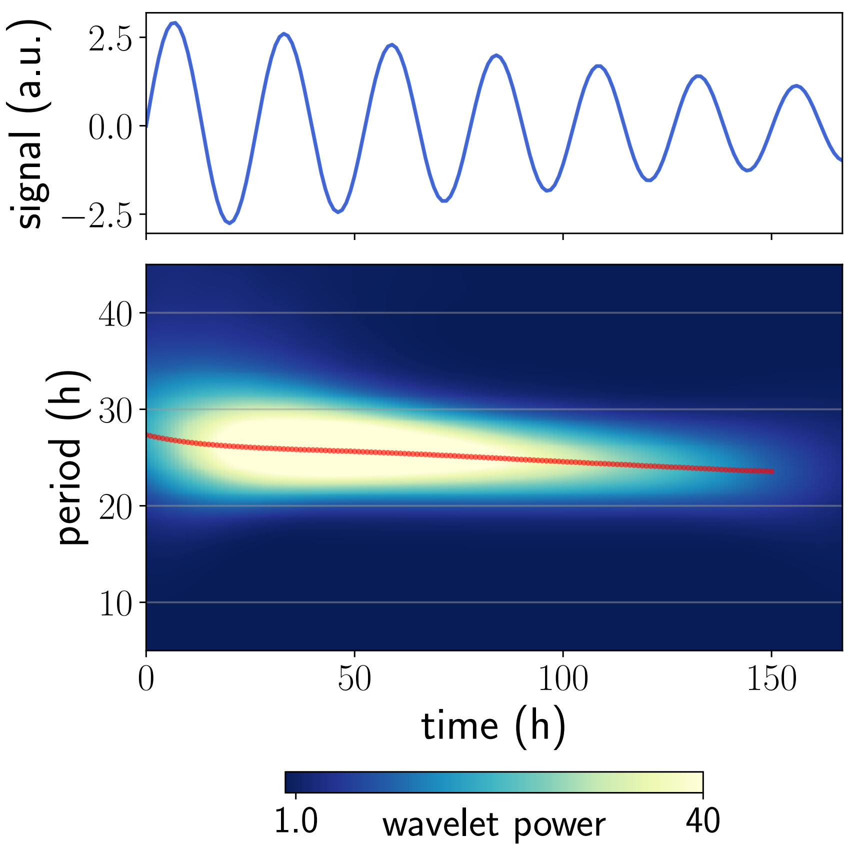
Ridge Evaluation



Readout



Ridge Evaluation



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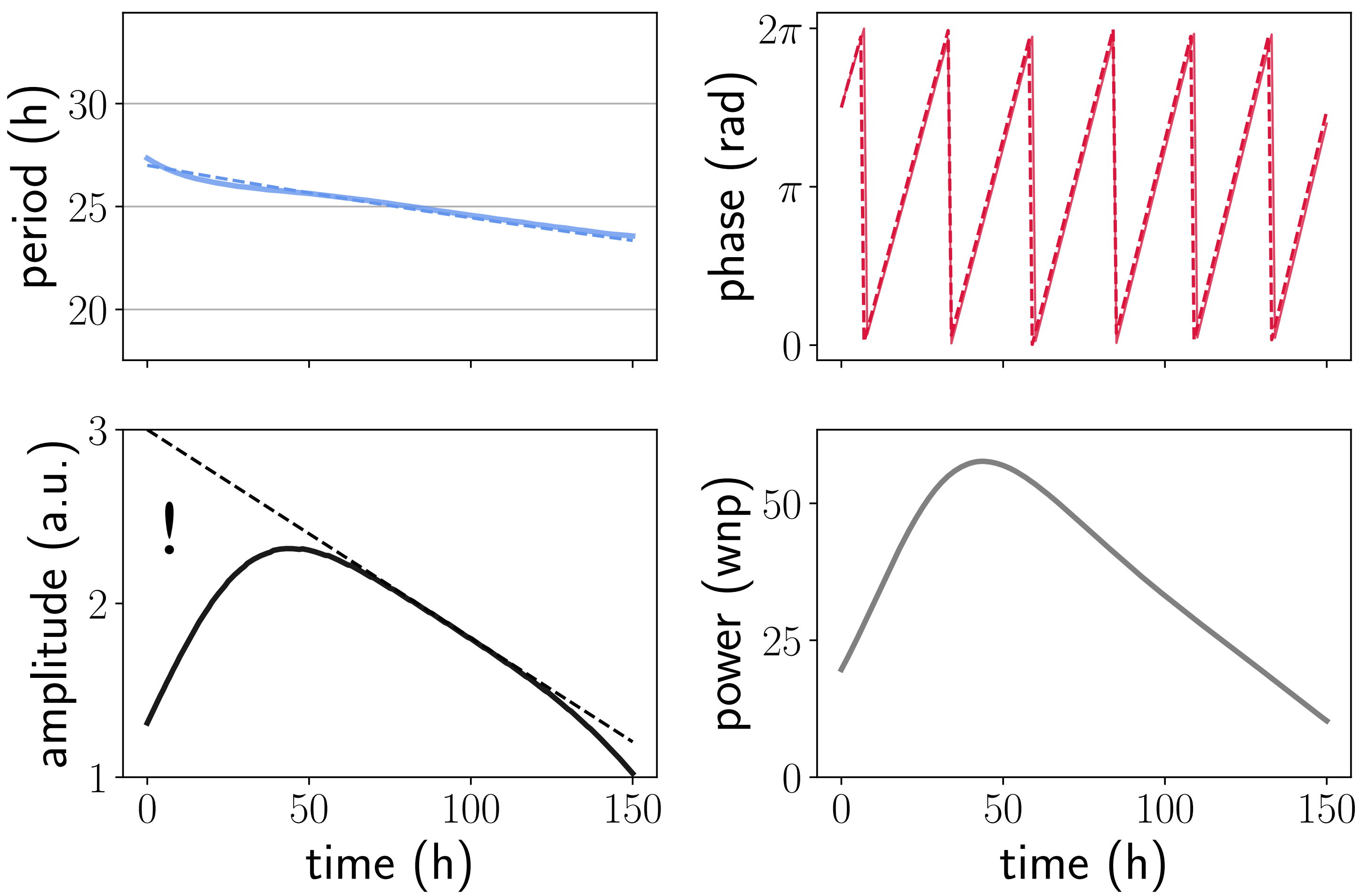
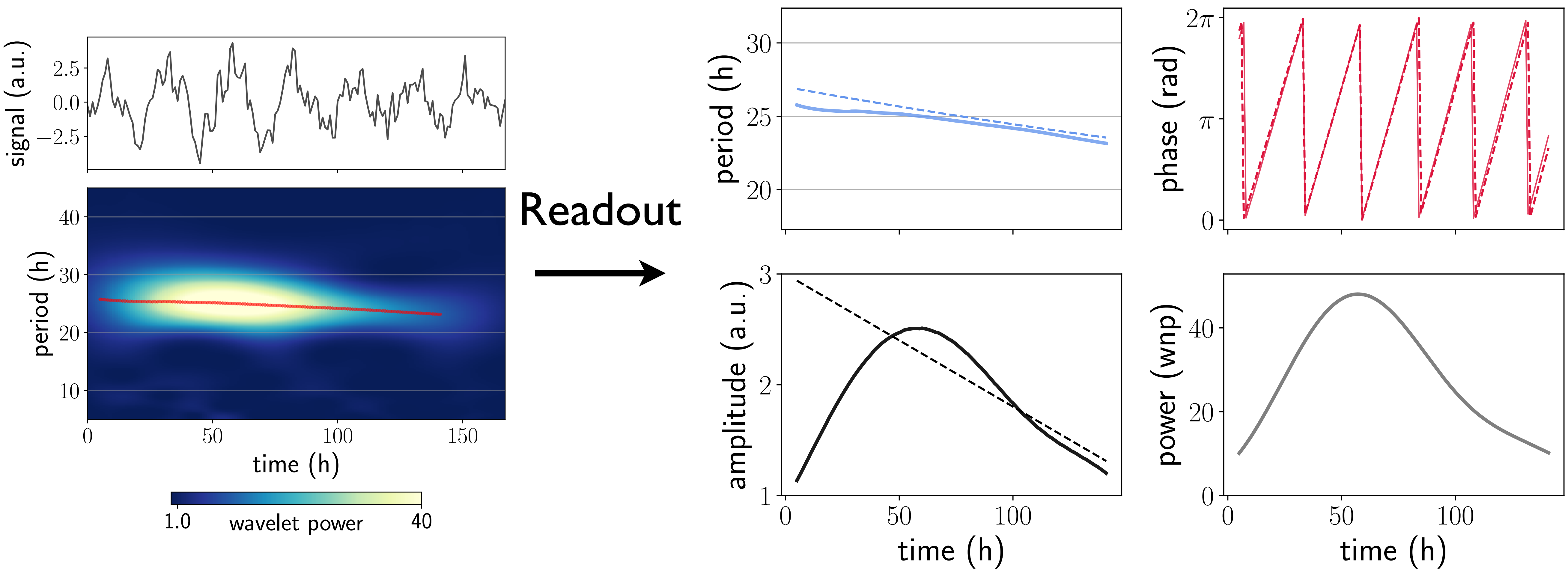


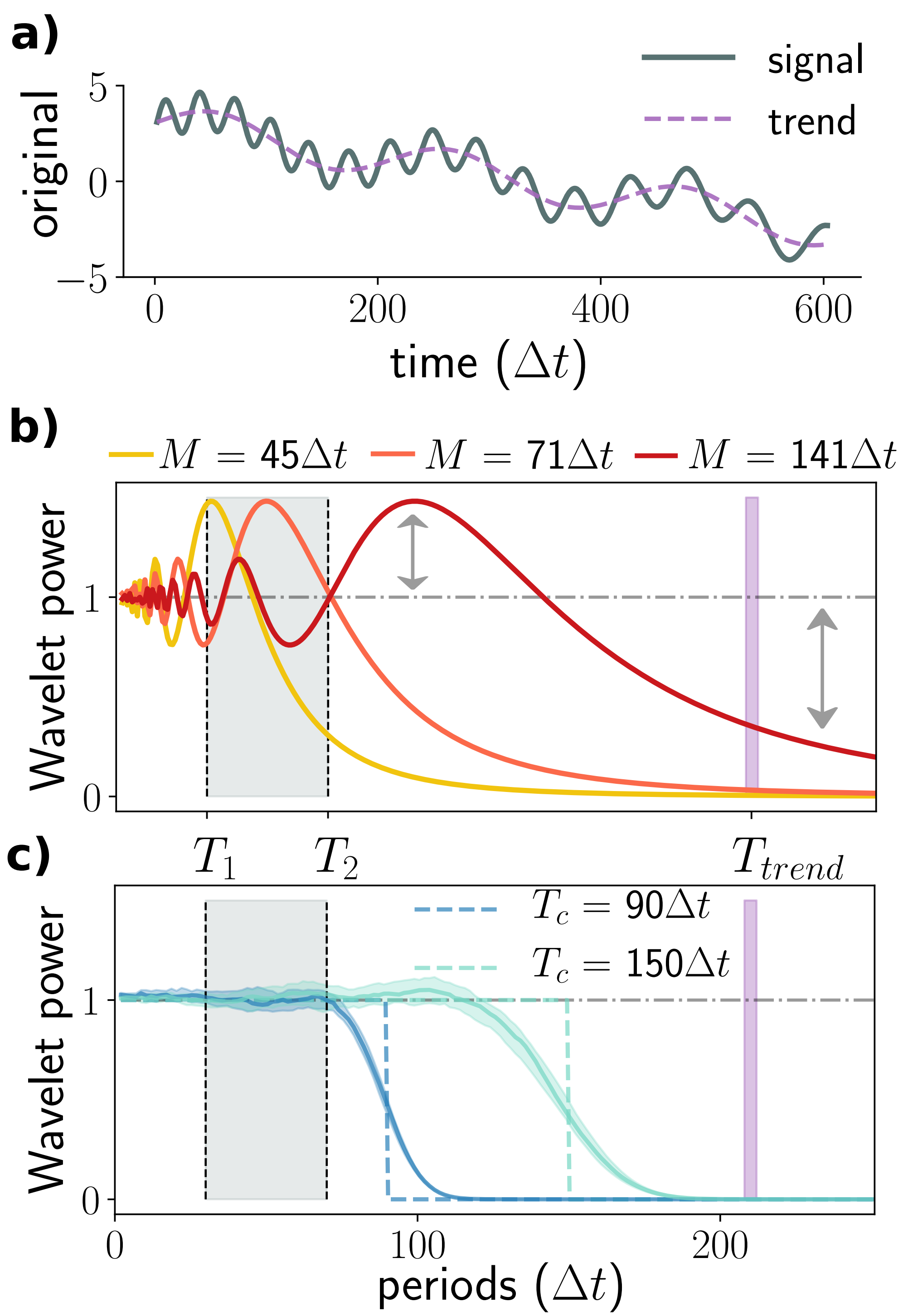
Edge effects of convolutions most prominent for amplitude estimation

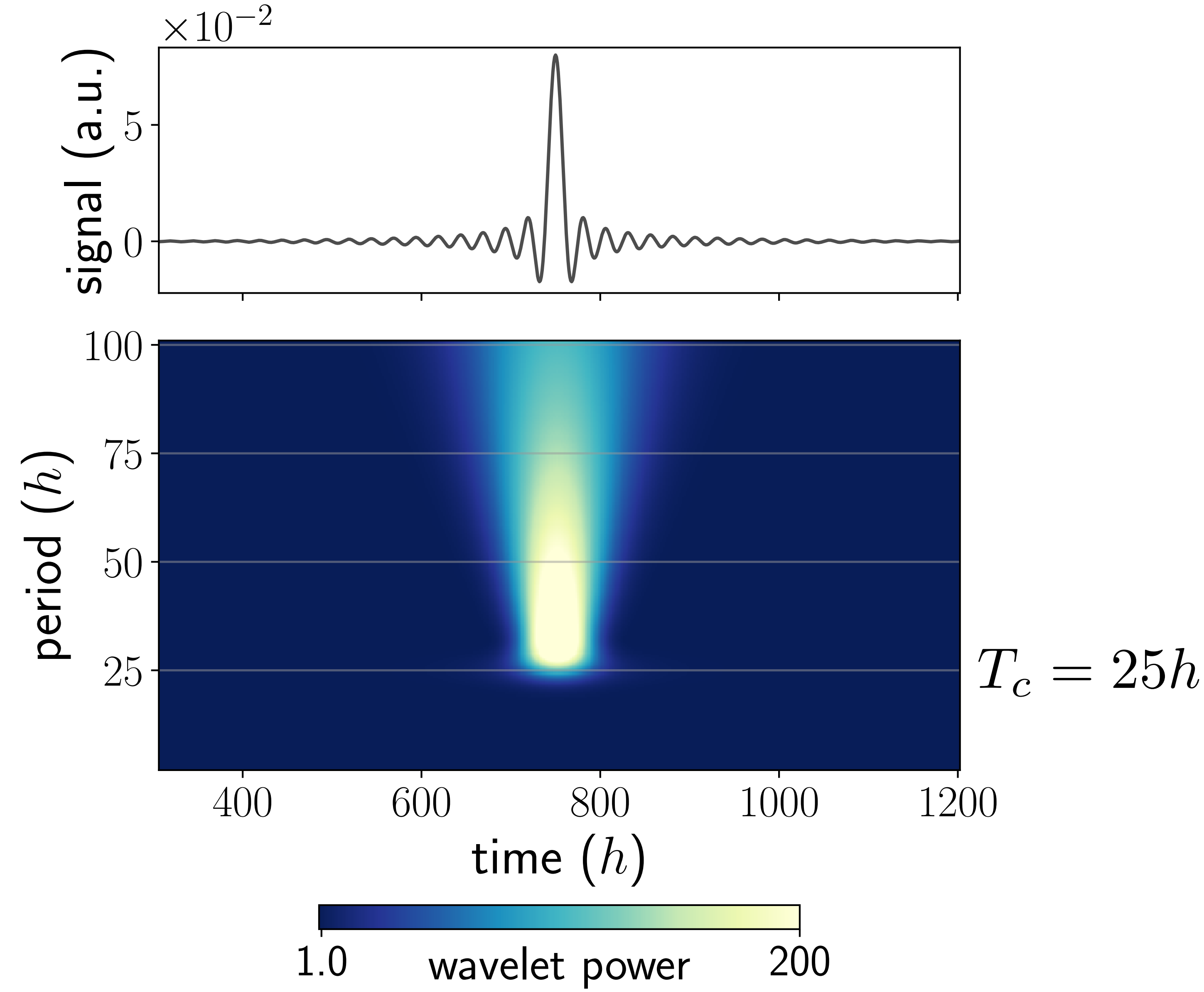
Noise Robustness



Wavelet analysis has a built-in
noise robustness!

Detrending with optimal Sinc Filter

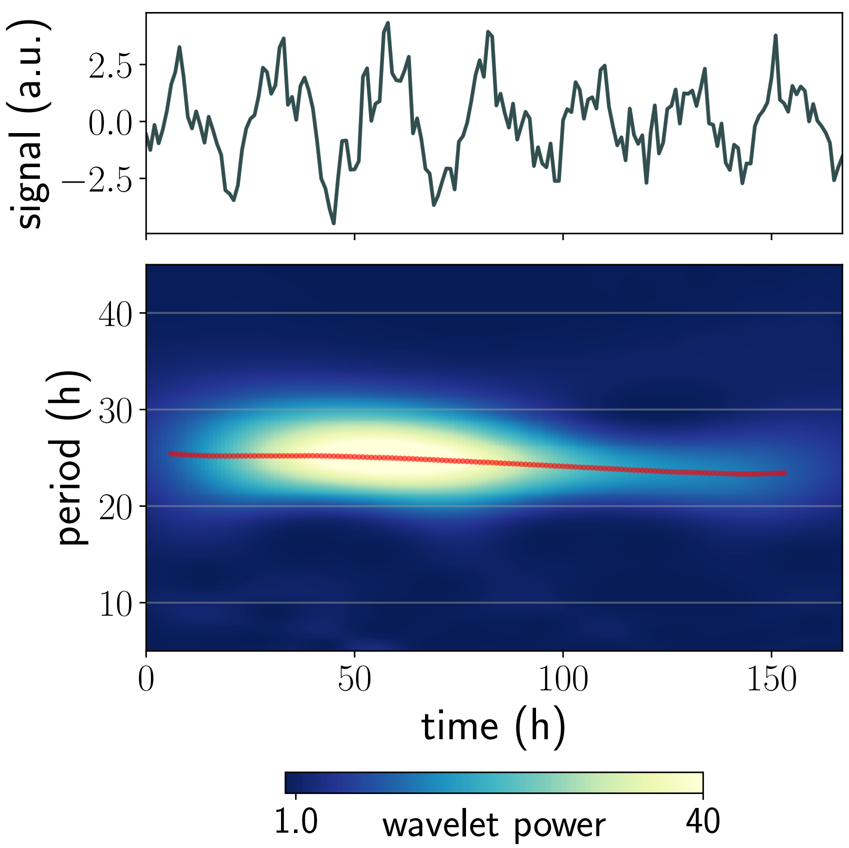




Cut-off period divides
pass- and stopband of the filter
without amplification or attenuation

$$1.5\,T_c \geq T_{max}$$

Noise + Trend



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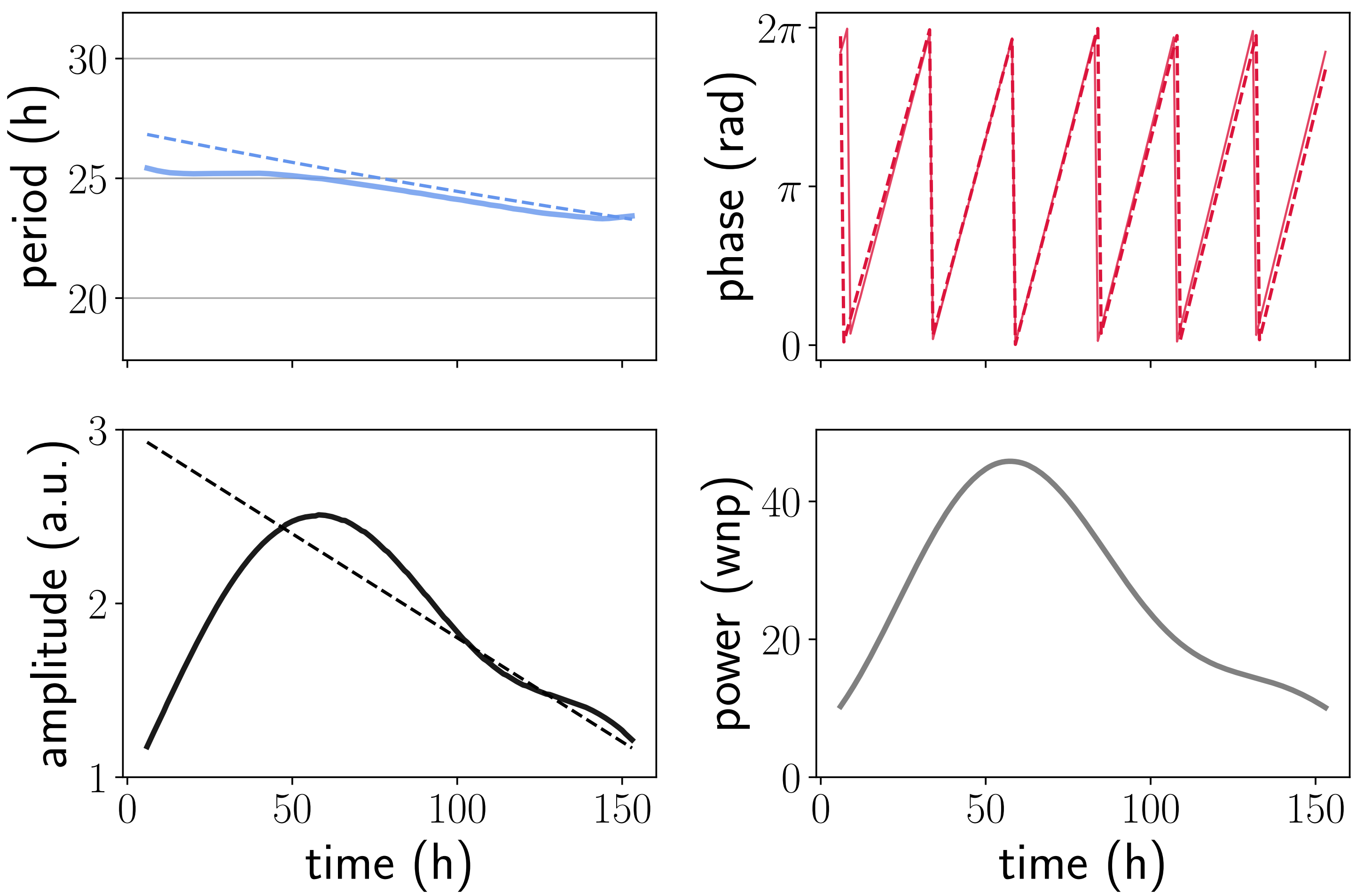
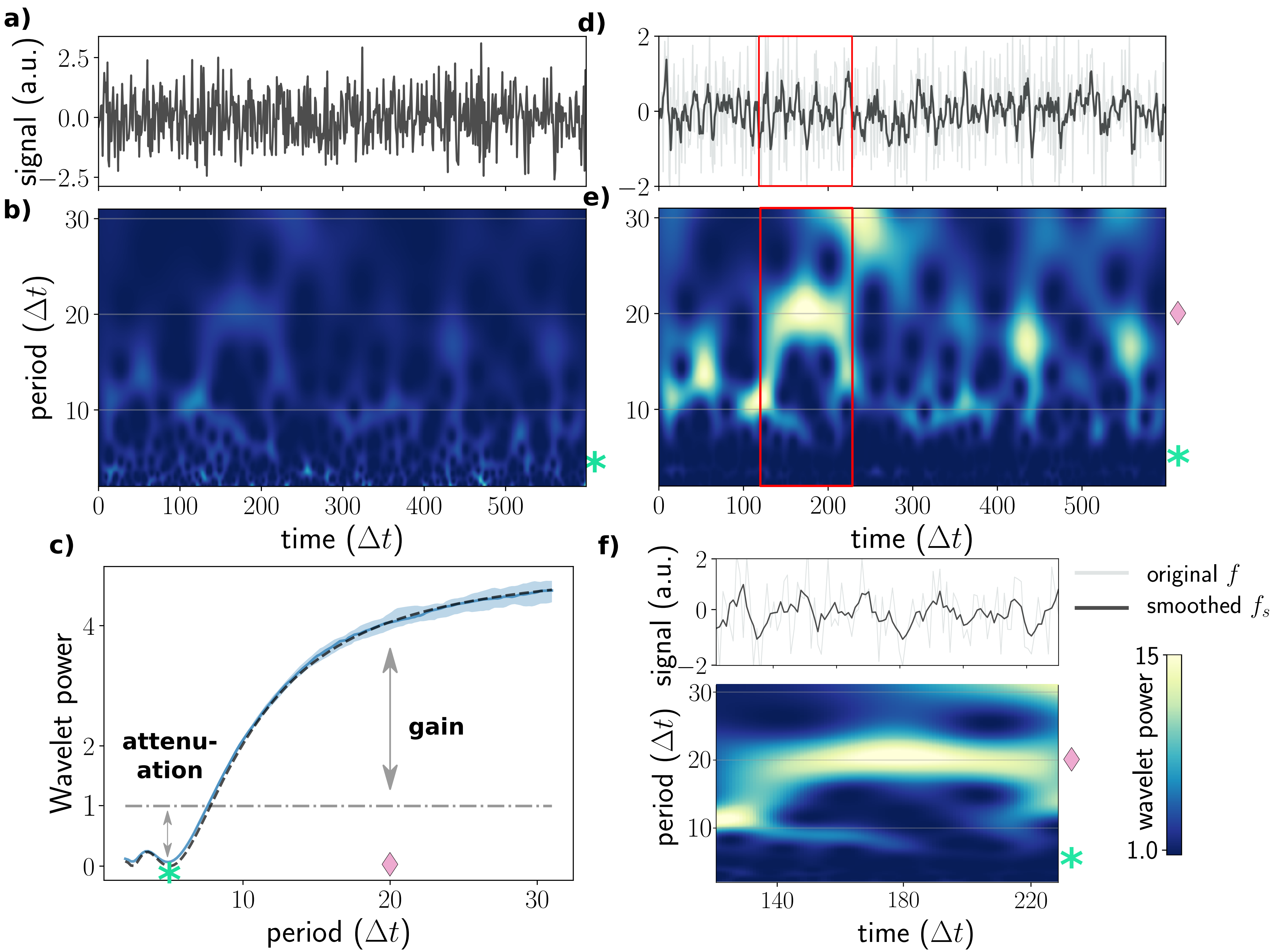


Detrending is practically perfect here

Smoothing Noise



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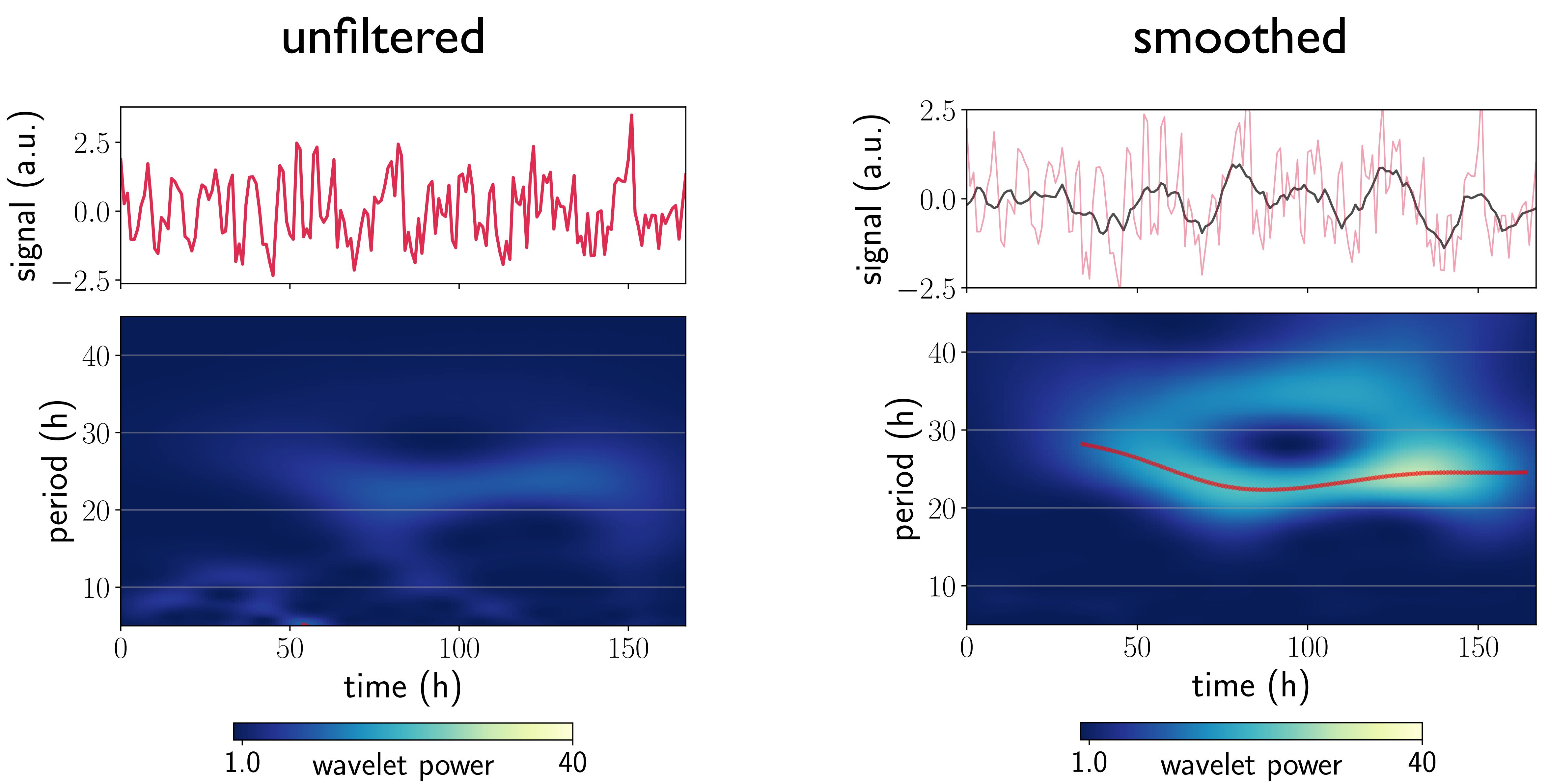

Wavelet analysis has a built-in
noise robustness: smoothing is not needed!

Wrapping up Time Series analysis