Available online at www.sciencedirect.com
ScienceDirect
Journal of the Franklin Institute 352 (2015) 3701–3715
Journal of The Franklin Institute
www.elsevier.com/locate/jfranklin

A novel communication-aware formation control strategy for dynamical multi-agent systems

Heng Li, Jun Peng*, Weirong Liu, Kai Gao, Zhiwu Huang

School of Information Science and Engineering, Central South University, Changsha 410000, China

Received 29 September 2014; received in revised form 6 March 2015; accepted 15 April 2015
Available online 5 May 2015

Abstract

In the classical studies of formation control, it is typically difficult to realize full potential of communication channels between agents, since the adopted communication links are typically assumed to be ideal or ideal within a certain communication range. In this paper, a more realistic communication channel model is considered and a new communication-aware formation control approach is proposed with the objective of optimizing communication performance of formation systems. A sufficient and necessary condition is found for feasible formation in realistic communication environments. Then a communication-aware formation control is proposed for multi-agent systems with switching topology. It is rigorously proved that the proposed algorithm can optimize the communication performance of formation systems. A simulation example is provided to illustrate the proposed design.
© 2015 The Franklin Institute. Published by Elsevier Ltd. All rights reserved.

1. Introduction

Formation control and wireless channel characterization have been developed separately for many years. Little knowledge about wireless channel has been used in formation control since communication links between agents are typically assumed ideal or ideal within a certain communication range [1,2]. In order to realize full potential of communication channels between

*Corresponding author. Tel.: +86 731 82539616; fax: +86 731 85640177.
E-mail addresses: liheng@csu.edu.cn (H. Li), pengj@csu.edu.cn (J. Peng), frat@csu.edu.cn (W. Liu), gaokai@csu.edu.cn (K. Gao), hzw@csu.edu.cn (Z. Huang).

http://dx.doi.org/10.1016/j.jfranklin.2015.04.008
0016-0032/© 2015 The Franklin Institute. Published by Elsevier Ltd. All rights reserved.

agents, it is of practical significance to model realistic channels for mobile agents and study how to control the formation group with realistic channel models.

Some early theoretical work in formation control generally assumes ideal communication channels in the formation design [3–5]. That is to say, there is no path loss between agents and every agent can communicate with all other agents effectively. More practical communication channels have been proposed in the past decade, and most of them are based on the communication range [6–13], where a binary channel model is applied, i.e., communication quality is assumed ideal (100%) within a certain radius of the transmitter and zero otherwise. In these studies, control laws are designed with a given communication constraint and do not rely on the real-time quality of communication channels. This fact, however, implies that agents lose the ability of reacting to the change of communication environments.

In recent years, much attention has been paid to modeling realistic communication channels for mobile agents in the control and communication community [14–23]. This interest is motivated by the vision of a multi-agent robotic network cooperatively adapting and learning in harsh unknown environments to achieve a common goal in the near future [14]. Many wireless link metrics, including bit error rate [2], received signal power [14,16], outage probability [15,21], transmission rate [17,19,20,22,23] and received channel to noise ratio (CNR) [18] have been examined in mobile agent networks for modeling communication channels between agents. In a realistic communication setting, the channel quality between agents is closely related to agents' positions or their relative distances [19]. Since the channel quality between agents can be measured or estimated by agents locally, communication-aware controllers are designed for agents by using channel quality as feedback instead of position or distance information.

Inspired by this new advance in modeling communication channels for mobile agents, this paper aims to study formation control of multi-agent systems in a practical communication environment. We model the communication channel between agents using reception probability, a realistic channel metric in the wireless communication theory [24]. In the proposed channel model, the channel quality is not assumed to be ideal or ideal within the communication range, but attenuating with the increase of propagation distances, which leads to a more accurate description of physical signal transmissions in a practical communication environment.

In classical formation control schemes, the desired distance between agents is typically predefined, and then gradient controllers are designed to keep the desired distance and maintain the formation [1]. The desired distance can be chosen arbitrarily by the designer as long as it is within the communication range, a value that is predefined as well. These controller design solutions may not be adequate for formation control in practical communication environments, where the communication condition of environments may change with time and space. The predefined desired distance may not always guarantee the optimal communication between agents in differing communication environments. A more reasonable idea is to design control laws by optimizing the communication performance of formation systems directly instead of keeping a predefined desired distance [11].

By summarizing our concerns on both the communication channel model and controller design method in existing formation control schemes, we attempt to propose a new communication-aware formation control strategy for multi-agent systems in a practical communication environment. The purpose of the design is to both maintain a stable formation and optimize the communication performance of the formation system. The contribution of this paper is twofold.

First, we model the communication channel between agents using reception probability, and bridge the gap between the communication channel model and the graph topology model. Then a communication performance indicator is proposed for formation systems in a practical communication environment, achieving a tradeoff between the antenna far-field and near-field communication. A sufficient and necessary condition is further provided for feasible formation in a practical communication environment.

Second, we design a new communication-aware formation control law to maintain the formation and optimize the communication performance. In the proposed formation control method, agents form a configuration that optimizes their communication performance without predefining a desired distance. The superiority of the proposed method over classical formation approaches in optimizing communication performance is rigorously proved with both theoretical analysis and simulation examples.

The reminder of the paper is organized as follows. In Section 2, preliminary knowledge about system model, graph theory, rigid formation and nonsmooth analysis is introduced. The communication channel modeling is presented in Section 3. In Section 4, a communication performance indicator is proposed for formation systems. In Section 5, a distributed communication-aware formation control law is proposed. Simulation examples are shown in Section 6. We conclude the paper in Section 7.

2. Preliminaries

2.1. System model

Consider a multi-agent system consisting of n agents. The dynamics of each agent in the group is given by

$$\dot{q}_i = u_i, \tag{1}$$

where $q_i, u_i \in R^2$ and $i \in \nu$, $\nu = \{1, 2, \ldots, n\}$. q_i and u_i denote the position and control input of the ith agent, respectively. We denote $u = [u_1^T, u_2^T, \ldots, u_n^T]^T$ as the control set and $q = [q_1^T, q_2^T, \ldots, q_n^T]^T$ as the formation set.

2.2. Graph theory

A graph G is a pair (ν,ε) that consists of a set of vertices $\nu = \{1, 2\ldots, n\}$ and edges $\varepsilon \subseteq \{(i,j)|i,j \in \nu, j \neq i\}$. The vertices in the graph denote the set of agents, and edges denote the information links between agents. The set of neighbors of agent i is defined as $N_i = \{j \in \nu|(i,j) \in \varepsilon\}$. The graph G is said to be undirected if $(i,j) \in \varepsilon \Leftrightarrow (j,i) \in \varepsilon$. The graph is said to be connected if there is a path between any two vertices of the graph. The length of any two vertices in a connected graph is no larger than $n-1$.

2.3. Rigid formation

Formation motions of a group of mobile agents in which distances between neighboring agents keep a prescribed desired distance are called rigid formation [1]. Many researchers investigate the distributed control of rigid formation using artificial potential fields. We denote r_{ij} as the Euclidean distance between agent i and agent j:

$$r_{ij} = \| q_i - q_j \|. \tag{2}$$

Then a formal definition of a potential function is given as follows:

Definition 1 (*Potential function*). A potential function $\Psi(r_{ij})$ between agent i and agent j is defined to have the following properties: (1) $\Psi(r_{ij})$ is a nonnegative function of r_{ij}; (2) $\Psi(r_{ij})$ is continuously differentiable; (3) $\Psi(r_{ij})$ reaches its strict minimum at $r_{ij}=r_\alpha$, i.e., $\Psi(r_\alpha)=0$ and $\Psi(r_{ij})>0$ for all $r_{ij}\neq r_\alpha$.

The potential function $\Psi(r_{ij})$ encodes a rigid formation with a desired distance r_α. A distributed rigid formation control law for system (1) can be designed as the negative gradient of its local potential functions:

$$u_i = -\nabla_{q_i}\left[\sum_{j\in N_i}\Psi(r_{ij})\right]. \tag{3}$$

2.4. Nonsmooth analysis

In this subsection, we introduce some tools from nonsmooth analysis that will used in the stability analysis of the proposed formation control.

We first review the Filippov solution of a differential equation with a discontinuous right-hand side.

Definition 2 (*Filippov solution [25]*). Consider the following differential equation in which the right-hand side can be discontinuous:

$$\dot{x} = f(x), \tag{4}$$

where $f: R^n \rightarrow R^n$ is measurable and essentially locally bounded. A vector function $x(\cdot)$ is called a Filippov solution of system (4) if it is absolutely continuous and for almost everywhere

$$\dot{x} \in K[f](x), \tag{5}$$

where

$$K[f](x) \equiv \overline{co}\left\{\lim_{x_i\rightarrow x} f(x_i)\middle| x_i \notin N\right\},$$

where $\overline{co}$ implies convex closure and N is a set of measure zero.

Clarke's generalized gradient describes the gradient of a function in the nonsmooth case.

Definition 3 (*Clarke's generalized gradient [26]*). For a locally Lipschitz function $V(x): R^n \rightarrow R$, Clarke's generalized gradient of V at x is defined as

$$\partial_x V = \overline{co}\left\{\lim \nabla_x V \middle| x_i \rightarrow x, x_i \notin N\right\}, \tag{6}$$

where N is the set of measure zero and the gradient of V is not defined. The generalized gradient $\partial_x V$ reduces to the classical gradient $\nabla_x V$ in the smooth case.

The following chain rule provides a calculus for the derivative of a regular function in the nonsmooth case. Regular functions include smooth functions and functions which can be written as the pointwise maximum of a set of smooth functions.

Lemma 1 (*Chain rule, Shevitz and Paden [27]*). *Let x be a Filippov solution to $\dot{x}=f(x)$ on an interval containing time t, and $V: R^n \rightarrow R$ be a Lipschitz and regular function. Then $V(x(t))$ is* *absolutely continuous, dV/dt exists almost everywhere and*

$$\frac{dV}{dt} \in^{a.e.} \dot{\tilde{V}} := \bigcap_{\xi \in \partial_x V} \xi^T K[f](x), \tag{7}$$

where "a.e." stands for "almost everywhere". The generalized time derivative $\dot{\tilde{V}}$ reduces to the classical derivative $\dot{V} = dV/dt$ in the case where the function is differentiable.

We will use the following nonsmooth version LaSalle's invariance principle to prove the convergence of the designed formation system.

Lemma 2 (*LaSalle's invariance principle, Shevitz and Paden [27]*). *Let Ω be a compact set such that every Filippov solution to the autonomous system $\dot{x} = f(x)$, $x(0) = x(t_0)$ starting in Ω is unique and remains in Ω for all $t \geq t_0$. Let $V : \Omega \to R$ be a time independent regular function such that $\tau \leq 0$ for all $\tau \in \dot{\tilde{V}}$ (if $\dot{\tilde{V}}$ is the empty set then this is trivially satisfied). Define $S = \{x \in \Omega | 0 \in \dot{\tilde{V}}\}$. Then every trajectory in Ω converges to the largest invariant set, E, in the closure of S.*

3. Communication channel modeling

Reception probability is an important wireless channel metric which allows for analysis of channel quality independent of a specific code design. An approximation for the reception probability of a SISO communication link is derived as follows [13]:

$$P(\alpha, \delta, \upsilon, r_0, r) = \exp\left(-\alpha\left(2^{\delta} - 1\right)\left(\frac{r}{r_0}\right)^{\upsilon}\right), \tag{8}$$

where α is a system parameter about antenna characteristics, δ is the required application data rate, υ is the path loss exponent, which depends on the physical environment (typically around $2 \sim 6$), r_0 is a reference distance for the antenna near-field, and r is the distance from the transmitter to the receiver.

The reception probability (8) evaluates the probability that the receiver can receive information accurately from the transmitter. In other words, it evaluates the probability that the transmitter can influence the receiver. We model the communication channel quality a_{ij} between agent i and agent j using the reception probability:

$$a_{ij} = \exp\left(-\alpha\left(2^{\delta} - 1\right)\left(\frac{r_{ij}}{r_0}\right)^{\upsilon}\right). \tag{9}$$

We then bridge the gap between the communication channel model and the graph topology model. The set of neighbors of agent i is now defined as

$$N_i = \left\{ j \in \nu | a_{ij} \geq P_T \right\}, \tag{10}$$

where P_T is a reception probability threshold. When the reception probability is less than P_T, agents just throw away packets they have received.

For the purpose of comparison with the classical binary channel model, we can also rewrite the neighbor set (10) as follows:

$$N_i = \left\{ j \in \nu \middle| r_{ij} \leq R \right\}, \tag{11}$$

where the communication range R is defined as $R = \arg_r \left\{ P(\alpha, \delta, \upsilon, r_0, r) = P_T \right\}$.

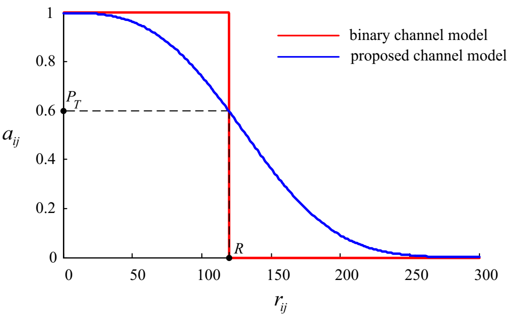

Fig. 1. The comparison of the proposed channel model with the classical binary channel model.

It is shown in Fig. 1 the comparison of the proposed channel model with the classical binary channel model. From Eq. (11) and Fig. 1, we can find there are two improvements in the proposed communication channel model. First, the communication range R is not chosen arbitrarily by the designer, but determined by application parameters (e.g., P_T and δ), antenna parameters (e.g., α and r_0) and environment parameters (e.g., υ). Second, the channel quality within the communication range is not assumed ideal but attenuating with the increase of transmission distances. Thus the proposed communication channel model provides a more accurate description of wireless channels in a practical communication environment.

4. Communication performance indicator

In this section, we design a communication performance indicator for formation systems to evaluate the communication performance of a formation system. The quality of a SISO communication channel is typically dominated by the path loss effect in the channel, where the reception probability of the receiver decreases with the increase of propagation distances [24]. However, in the formation control scenario, the path loss channel model is typically only valid in the antenna far-field, i.e., neighboring agents are far enough from each other. In the antenna near-field, the communication between agents suffers from severe mutual interference, which will degrade the communication performance of formation systems [28]. The accurate communication condition in the antenna near-field can be obtained by complicated empirical measurements. However, for general tradeoff analysis of various system designs, it is more practical to use a simple approximation model instead of resorting to complicated empirical models [24]. Thus we approximate the antenna near-field communication with the following simple model:

$$g_{ij} = \frac{r_{ij}}{\sqrt{r_{ij}^2 + r_0^2}}. \tag{12}$$

It can be concluded from Eq. (12) that when $r_{ij} \to 0$, the channel quality $g_{ij} \to 0$, which characterizes the interference effect in the antenna near-field; while when r_{ij}/r_0, $g_{ij} \to 1$, implying that the interference effect can be ignored in the antenna far-field. The model (12) is a simple but effective model that captures the essence of signal propagation phenomena in the antenna near-field.

We propose a communication performance indicator for formation systems by fully considering the path loss effect in the antenna far-field and the interference effect in the antenna

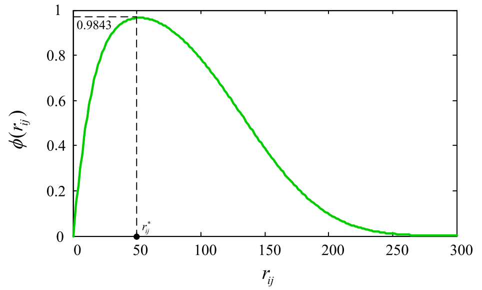

Fig. 2. The proposed communication performance indicator $\phi(r_{ij})$ for formation systems.

near-field. The communication performance indicator is designed as follows:

$$\phi\left(r_{ij}\right) = \frac{r_{ij}}{\sqrt{r_{ij}^2 + r_0^2}} \cdot \exp\left(-\alpha\left(2^{\delta}-1\right)\left(\frac{r_{ij}}{r_0}\right)^{\upsilon}\right). \tag{13}$$

Remark 1. The design of communication performance indicator (13) is mainly inspired by a "tradeoff philosophy" which is typically adopted in mobile communication systems to evaluate a comprehensive performance regarding two or more aspects of a system; see e.g., [15,22,23]. The aim of such a design is to ensure that the optimal communication performance is achieved at a tradeoff distance between agents, where neighboring agents are neither too far away to weaken the path loss effect nor too close to avoid the interference and collision.

It is shown in Fig. 2 the proposed communication performance indicator $\phi\left(r_{ij}\right)$ for formation systems. It can be found that the communication performance decays severely in the antenna near-field due to the mutual interference between agents, and decreases slowly for r_{ij}/r_0 due to the path loss effect. This fact implies that the proposed communication indicator conforms well to the wireless propagation phenomena of swarm systems [28]. From Fig. 2, it can be observed that the indicator (13) reaches its maximum at a certain distance r_{ij}^*. It is an important observation that provides a possibility of designing a communication-aware formation control law to maintain a configuration that optimizes the communication performance of formation systems. This observation is rigorously proved in the following theorem.

Theorem 1. *The communication performance indicator* (13) *is a continuously differentiable function with a strict maximum* $\phi^* = \phi\left(r_{ij}^*\right)$, *where* $r_{ij}^* \in \left(0, r_0/\left(\alpha\upsilon\left(2^{\delta}-1\right)\right)^{1/\upsilon}\right)$.

Proof. It is straightforward that the communication performance indicator $\phi\left(r_{ij}\right)$ is continuously differentiable with respect to r_{ij}. For concision, we rewrite Eq. (13) as

$$y(x) = \frac{x}{\sqrt{x^2 + r_0^2}} \cdot \exp\left(-\beta\left(\frac{x}{r_0}\right)^{\upsilon}\right), \tag{14}$$

where $x = r_{ij} \in [0, \infty)$, $y(x) = \phi(r_{ij})$, and $\beta = \alpha\left(2^{\delta}-1\right) > 0$.

Then the derivative of y with respect to x can be derived as

$$\rho(x) = \frac{dy}{dx} = \frac{-\beta\upsilon x^{\upsilon+2} - \beta\upsilon r_0^2 x^{\upsilon} + r_0^{\upsilon+2}}{\sqrt{(x^2 + r_0^2)^3}} \cdot \exp\left(-\beta\left(\frac{x}{r_0}\right)^{\upsilon}\right), \tag{15}$$

where $\sqrt{(x^2+r_0^2)^3}>0$ and $\exp\left(-\beta\left(x/r_0\right)^{\upsilon}\right)>0$ for all $x\in[0,\infty)$. So whether $y(x)$ has an extremum point $y^*=\left\{y\middle|dy/dx=0\right\}$ is totally determined by the function

$$f(x)=-\beta\upsilon x^{\upsilon+2}-\beta\upsilon r_0^2x^{\upsilon}+r_0^{\upsilon+2}. \tag{16}$$

The derivative of $f(x)$ with respect to x is derived as

$$\frac{df(x)}{dx}=-x^{\upsilon-1}\left[\beta\upsilon^2r_0^2+\beta\upsilon(\upsilon+2)x^2\right]<0, \quad \forall x>0. \tag{17}$$

Thus $f(x)$ is a continuous and strictly decreasing function. Since $f(0)=r_0^{\upsilon+2}>0$ and $f\left(r_0/(\beta\upsilon)^{1/\upsilon}\right)=-r_0^{\upsilon+2}/(\beta\upsilon)^{2/n}<0$, there must be one and only one $\varsigma\in\left(0,r_0/(\beta\upsilon)^{1/\upsilon}\right)$ that lets $f(\varsigma)=0$. Then we have

$$\begin{cases}\rho(x)>0, & \forall x\in[0,\varsigma),\\ \rho(x)=0, & \forall x=\varsigma,\\ \rho(x)<0, & \forall x\in(\varsigma,\infty),\end{cases} \tag{18}$$

which means that $y(x)$ reaches its strict maximum at $x=\varsigma\in\left(0,r_0/\left(\alpha\upsilon\left(2^{\delta}-1\right)\right)^{1/\upsilon}\right)$. This completes the proof.□

Theorem 1 implies that if neighboring agents in a formation keep the distance r_{ij}^*, the pairwise communication performance of them can be optimized. Then the desired formation q^* that optimizes the communication performance (13) is defined as

$$q^*=\left\{q\middle|r_{ij}=r_{ij}^*,\ \forall(i,j)\in\varepsilon\right\}. \tag{19}$$

Before the formation control law is designed, we first evaluate if the desired formation (19) can be achieved in a specific communication setting. The definition of a feasible formation is given as follows.

Definition 4 (*Feasible formation*). The desired formation q^* is called feasible if $r_{ij}^*\geq r_0$ holds in the given communication setting.

Remark 2. When the desired distance r_{ij}^* is less than the antenna near-field reference distance r_0, the desired formation (19) is considered unfeasible. This is because in the antenna near-field, the communication between agents becomes unreliable. Maintaining a desired formation in the antenna near-field increases the risk of collisions between agents.

The following theorem provides a sufficient and necessary condition for feasible formation in a given communication setting.

Theorem 2. *The desired formation (19) is called feasible only when the path loss exponent υ and system parameters α, δ satisfy $\upsilon\leq 1/2\alpha\left(2^{\delta}-1\right)$.*

Proof. From Definition 4, the sufficient and necessary condition for feasible formation is $r_{ij}^*\geq r_0$. Based on Eq. (18), we have $\rho(r_0)\geq 0$, specifically,

$$f(r_0)=-\beta\upsilon r_0^{\upsilon+2}-\beta\upsilon r_0^{\upsilon+2}+r_0^{\upsilon+2}\geq 0. \tag{20}$$

Then we have

$$\upsilon \leq \frac{1}{2\alpha\left(2^{\delta}-1\right)}. \tag{21}$$

This completes the proof.□

5. Communication-aware formation controller

In this section, we design a communication-aware formation controller to optimize the communication performance and maintain the desired formation. The communication-aware formation control law is designed and analyzed with tools from artificial potential fields and nonsmooth techniques, respectively.

We define a pairwise artificial potential function $\psi(r_{ij})$ by

$$\psi\left(r_{ij}\right) = \phi^{*} - \phi\left(r_{ij}\right), \quad \forall (i,j) \in \varepsilon. \tag{22}$$

The potential function (22) only evaluates the interaction between neighboring agents. When the topology switches with Eq. (10), some edges may be lost or created. Thus we define a new potential function to evaluate the interaction between any pairs of agents regardless of whether they are neighbors or not. The pairwise potential function $\psi_t(r_{ij})$ is defined as

$$\psi_t(r_{ij}) = \begin{cases} \psi(r_{ij}), & \forall (i,j) \in \varepsilon, \\ \psi(R) & \text{otherwise}. \end{cases} \tag{23}$$

The potential function $\psi_t(r_{ij})$ keeps fixed for $r_{ij} \geq R$, implying that when edge (i,j) is lost, agents i and agent j will not influence each other any more. It is worth noting that $\psi_t(r_{ij})$ is not necessarily differentiable at the transition point $r_{ij} = R$.

The collective potential function of the formation group is denoted as

$$W(q) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\psi_t\left(r_{ij}\right). \tag{24}$$

Then a communication-aware formation controller is designed as

$$u_i = -\nabla_{q_i}\left[\sum_{j \in N_i}\psi(r_{ij})\right] = \nabla_{q_i}\left[\sum_{j \in N_i}\phi(r_{ij})\right]. \tag{25}$$

The formation controller (25) implies that agents move in the direction of maximizing the communication performance of neighboring agents.

The gradient of $\phi(r_{ij})$ is computed as

$$\nabla_{q_i}\phi(r_{ij}) = \rho\left(r_{ij}\right) \cdot e_{ij}, \tag{26}$$

where $e_{ij} = \left(q_i - q_j\right)/r_{ij}$.

Then the communication-aware formation controller (25) can be further written as

$$u_i = \sum_{j \in N_i}\rho\left(r_{ij}\right) \cdot e_{ij} \tag{27}$$

Note that when the topology switches, the right-hand side of potential function $W(q)$ may not be smooth. In what follows, we analyze the convergence of the formation system using nonsmooth techniques.

Then the following theorem holds.

Theorem 3. *Consider the multi-agent system* (1) *evolves under the control law* (25) *in a practical communication environment. Let* $\Omega_c = \{q|W(q) \le c\}$ *for* $0 < c < \phi^*$ *be a level set of W* (*q*). *Assume* Theorem 2 *holds and the graph is connected. Then for any solution starting in* Ω_c, *the following statements hold*:

(1) *the closed-loop formation system is stable*;
(2) *collisions between agents can be avoided*;
(3) *the formation q converges to the desired formation* q^* *asymptotically.*

Proof. We first show that the level set Ω_c is compact with respect to relative positions of agents. It is straightforward that Ω_c is closed by continuity and the boundedness follows from connectivity. Specifically, from $W(q) \le c$ we have $\psi\left(r_{ij}\right) \le c$ for all $(i,j) \in \varepsilon$. This implies that there is a τ, where $0 < \tau < \infty$, such that $r_{ij} \le \tau$ for all $(i,j) \in \varepsilon$. Since the graph is connected, we have $0 \le r_{ij} \le (n-1)\tau$ for all $i,j \in \nu$, which implies interagent distances between any pair of agents are finite. Thus the level set Ω_c is compact.

Clarke's generalized gradient of $W(q)$ is denoted as

$$\partial_q W(q) = \left[\sum_{j=1}^{n} \partial_{q_1^T} \psi_t\left(r_{1j}\right), \ldots, \sum_{j=1}^{n} \partial_{q_n^T} \psi_t\left(r_{nj}\right) \right]^T. \tag{28}$$

Based on the chain rule in Lemma 1, we have the generalized time derivative as

$$\dot{\tilde{W}}(q) = \bigcap_{\xi \in \partial_q W} \left(\xi^T K[-\partial_q W](q)\right) = -\sum_{i=1}^{n}\sum_{j=1}^{n} \bigcap_{\xi_{ij}} \xi_{ij}^T \xi_{ij}, \tag{29}$$

where $\xi_{ij} \in \partial_{q_i} \psi_t(r_{ij})$.

At R where $\psi_t(r_{ij})$ is not differentiable, $\partial_{r_{ij}} \psi_t(R)$ is empty, and thus, $\partial_{q_i} \psi_t(R)$ is empty. Then we have

$$\dot{\tilde{W}}(q) = -\sum_{i=1}^{n}\sum_{j \in N_i} \zeta_{ij}^T \zeta_{ij} \le 0, \tag{30}$$

where $\zeta_{ij} = \nabla_{q_i} \psi(r_{ij})$ for all $(i,j) \in \varepsilon$.

Now compactness of Ω_c and Eq. (30) guarantee that the closed loop system is stable. Then the first statement holds.

From LaSalle's invariance principle in Lemma 2, we know that the system converges to the largest invariant subset $E = \{q|\dot{\tilde{W}}(q) = 0\}$. Specifically, from Eqs. (26) and (30), the largest invariant subset E can be further rewritten as

$$E = \left\{q \middle| \rho\left(r_{ij}\right) e_{ij} = 0, \ \forall (i,j) \in \varepsilon\right\}. \tag{31}$$

We prove the second statement by contradiction. We denote the set of initial condition as

$$\Phi_0 = \left\{q(0) \middle| \, \| q_i(0) - q_j(0) \| > 0, \forall i,j \in \nu, i \ne j\right\}. \tag{32}$$

For every initial condition $q(0) \in \Phi_o$, the generalized time derivative of W remains non-positive for all $t \ge 0$, thus we have

$$W(q(t)) \le W(q(0)) \le c < \phi^*. \tag{33}$$

When $r_{ij}=0$, from Eq. (22) we have $\psi_t(0)=\phi^*$. Assume there exists at least a pair of agents k, l collide at time $t=t_m$, i.e. $q_k(t_m)=q_l(t_m)$. For all $t\geq 0$, we have

$$\begin{aligned} W(q(t)) &= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\psi_t(r_{ij}) \\ &= \psi_t(r_{kl}) + \frac{1}{2}\sum_{i\in\nu/\{k,l\}}\sum_{j\in\nu/\{k,l\}}\psi_t(r_{ij}) \\ &\geq \psi_t(r_{kl}). \end{aligned} \tag{34}$$

Thus we have

$$W(q(t_m)) \geq \psi_t(0) = \phi^*, \tag{35}$$

which is in contradiction with Eq. (33). Hence Φ_0 is invariant for the trajectory of the closed loop system and collision avoidance is achieved, i.e., $q_i(t)-q_j(t)\neq 0$ for all $i\neq j$ as $t\to\infty$. Now the second statement holds.

Since $q_i(t)-q_j(t)\neq 0$, we have $e_{ij}\neq 0$. Then from the largest invariant subset (31), we obtain $\rho(r_{ij})=0$. From Theorem 1, we conclude that the formation q converges to the desired formation (19). This completes the proof.□

Theorem 3 implies that when the desired formation q^* is achieved, the communication performance is optimized. The following corollary proves that compared with the classical rigid formation control, the proposed communication-aware formation control can optimize the communication performance of formation systems.

Corollary 1. *Compared with the rigid formation control law* (3), *the communication-aware formation control law* (25) *can optimize the communication performance* (13) *of formation systems in a practical communication environment.*

Proof. For the communication-aware formation control law (25), $\phi(r_{ij})\to\phi^*$ as $q\to q^*$ is guaranteed for all $(i,j)\in\varepsilon$ in Theorem 3. While for the rigid formation control law (3), assume that the formation is maintained, then we have $r_{ij}\to r_\alpha$ for all $(i,j)\in\varepsilon$, where r_α is prescribed. From Theorem 1 we know that $\phi(r_{ij})$ reaches its strict maximum ϕ^* at r_{ij}^*, we have $\phi(r_\alpha)=\phi^*$ for $r_\alpha=r_{ij}^*$ and $\phi(r_\alpha)<\phi^*$ for any $r_\alpha\neq r_{ij}^*$. Since the optimal communication distance r_{ij}^* is determined by system parameters and cannot be obtained a priori, it is almost impossible to predefine $r_\alpha=r_{ij}^*$ by the designer. Then $\phi(r_{ij}^*)>\phi(r_\alpha)$ holds for most cases. This completes the proof.□

6. Simulation example

In this section, we provide a simple simulation example to illustrate the proposed formation control method. The following parameters of the communication setting are used in the simulation: $\alpha=10^{-5}$, $\delta=2$, $n=3$, $r_0=5$, $P_T=93.2\%$.

Consider a group of 7 agents evolving in a practical communication environment with the given communication setting. The initial positions of 7 agents are given by $x_1=[-5,14]^T$, $x_2=[-5,-19]^T$, $x_3=[0,0]^T$, $x_4=[35,-4]^T$, $x_5=[68,0]^T$, $x_6=[72,13]^T$, $x_7=[72,-18]^T$.

The initial topology of 7 agents in the given communication environment is shown in Fig. 3. The circular node represents an agent. The blue line represents the initial communication link

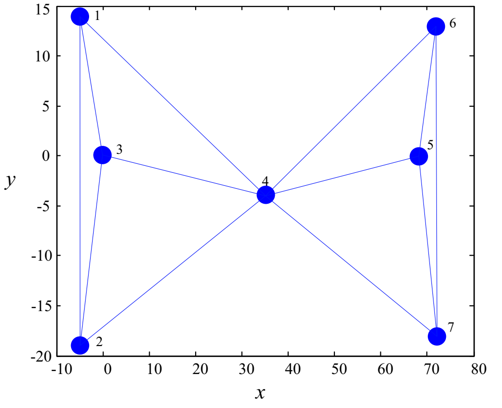

Fig. 3. The initial topology of 7 agents. The node represents an agent; the blue line represents the initial communication link between agents. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

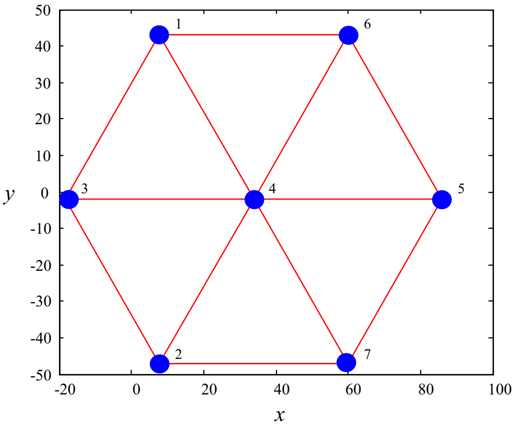

Fig. 4. The final topology of 7 agents. The node represents an agent; the red line represents the optimal communication link between agents. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

between agents. Then agents are steered by control law (25) to evolve in the given communication environment. The final topology of 7 agents is shown in Fig. 4. We can see that some initial links are lost and some new links are created. Moreover, the group converges to a stable formation. The evolution of 7 agents is further shown in Fig. 5.

In order to compare the proposed formation control method with the classical rigid formation control method, we define an average communication performance indicator as

$$J_n = \frac{\sum_{i=1}^{n} \sum_{j \in N_i} \phi(r_{ij})}{2n|N_i|},$$

where $|N_i|$ is the number of neighbors of agent i. The indicator J_n evaluates the average communication performance of the formation system.

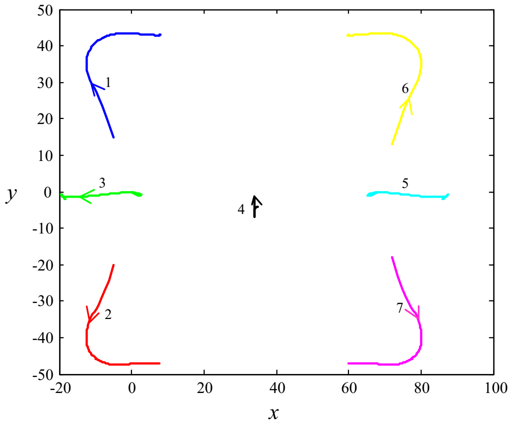

Fig. 5. The evolution of 7 agents in the given communication environment. The arrow represents the moving direction of agents.

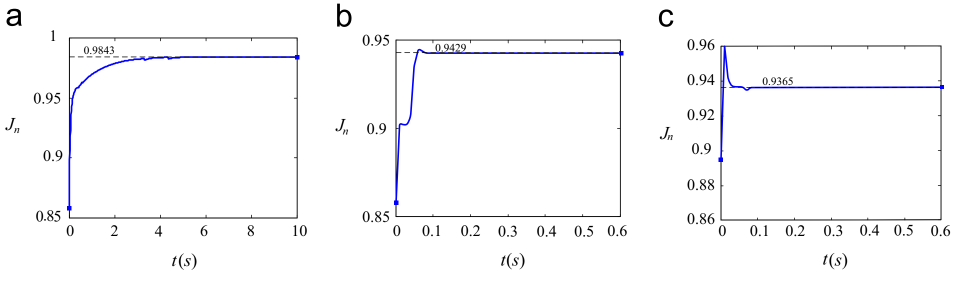

Fig. 6. The comparison of the communication-aware formation control with the classical rigid formation control. (a) Communication-aware formation control. (b) Rigid formation control with $r_\alpha = 35$. (c) Rigid formation control with $r_\alpha = 75$.

It is shown in Fig. 6 the comparison of the proposed communication-aware formation control with the classical rigid formation control. Fig. 6(a) shows the average communication performance J_n of the formation system driven by the communication-aware formation control law keeps increasing to the maximum $\phi^* = 98.43\%$, implying that agents evolve in the direction of maximizing the communication performance of the formation system. In contrast, it is shown that the rigid formation control suffers from performance loss due to the path loss effect in the antenna far-field. Simulation results show that the proposed communication-aware formation scheme can optimize the communication performance of formation systems.

7. Conclusions

In this paper, we propose a new communication-aware formation control strategy for multi-agent systems in a practical communication environment. The communication channel between agents is modeled by reception probability, which provides a more accurate description of SISO channels between agents. Then we design a communication performance indicator for formation systems, achieving a tradeoff between the antenna far-field communication and antenna near-field communication. Communication-aware formation control laws are designed by optimizing the communication performance directly without keeping a predefined desired distance. The stability and convergence of the proposed controller is analyzed with tools from nonsmooth techniques. The superiority of the proposed communication-aware control strategy in optimizing communication performance is rigorously proved with theoretical analysis. Moreover, simulation results are provided to illustrate the effectiveness of the proposed design. Future work will focus on the probabilistic channel modeling for mobile agents and adaptive controller design for formation systems.

Acknowledgment

The authors would like to thank the anonymous reviewers for their constructive comments that improved both the presentation and result of this work. This work is supported by National Nature Science Foundation of China (Grant nos. 61202342, 61379111, 61402538, 61403424) and Hunan Provincial Innovation Foundation for Postgraduate.

Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jfranklin.2015.04.008.

References

[1] J. Baillieul, P.J. Antsaklis, Control and communication challenges in networked real-time systems, Proc. IEEE 95 (1) (2007) 9–28.
[2] Y. Yan, Y. Mostofi, Robotic router formation in realistic communication environments, IEEE Trans. Robot. 28 (4) (2012) 810–827.
[3] F. Giulietti, L. Pollini, M. Innocenti, Autonomous formation flight, IEEE Control Syst. 20 (6) (2000) 34–44.
[4] N.E. Leonard, E. Fiorelli, Virtual leaders, artificial potentials and coordinated control of groups, in: 40th IEEE Conference on Decision and Control, 2001, pp. 2968–2973.
[5] V. Gazi, K.M. Passino, Stability analysis of swarms, IEEE Trans. Autom. Control 48 (4) (2003) 692–697.
[6] H.G. Tanner, A. Jadbabaie, G.J. Pappas, Flocking in fixed and switching networks, IEEE Trans. Autom. Control 52 (5) (2007) 863–868.
[7] D.V. Dimarogonas, K.J. Kyriakopoulos, A connection between formation infeasibility and velocity alignment in kinematic multi-agent systems, Automatica 44 (10) (2008) 2648–2654.
[8] Z. Qu, J. Wang, R.A. Hull, Cooperative control of dynamical systems with application to autonomous vehicles, IEEE Trans. Autom. Control 53 (4) (2008) 894–911.
[9] D.V. Dimarogonas, K.H. Johansson, Stability analysis for multi-agent systems using the incidence matrix: quantized communication and formation control, Automatica 46 (4) (2010) 695–700.
[10] Y. Cao, W. Ren, Distributed coordinated tracking with reduced interaction via a variable structure approach, IEEE Trans. Autom. Control 57 (1) (2012) 33–48.
[11] H. Li, J. Peng, W. Liu, J. Wang, J. Liu, Z. Huang, Flocking control for multi-agent systems with communication optimization, in: 2013 American Control Conference, 2013, pp. 2056–2061.
[12] J. Wang, M. Obeng, T. Yang, G. Staskevich, B. Abbe, Formation control of multiple nonholonomic mobile robots with limited information of a desired trajectory, in: 2014 IEEE International Conference on Electro/Information Technology, 2014, pp. 550–555.
[13] C. Li, Z. Qu, M.A. Ingram, Distributed extremum seeking and cooperative control for mobile communication, in: 50th IEEE International Conference on Decision, 2011, pp. 4510–4515.

[14] Y. Mostofi, A. Gonzalez-Ruiz, A. Gaffarkhah, D. Li, Characterization and modeling of wireless channels for networked robotic and control systems—a comprehensive overview, in: 2009 IEEE International Conference on Intelligent Robots and Systems, 2009, pp. 4849–4854.
[15] C. Li, Z. Qu, M.A. Weitnauer, Distributed extremum seeking and formation control for nonholonomic mobile network, Syst. Control Lett. 75 (2015) 27–34.
[16] A. Ghaffarkhah, Y. Mostofi, Communication-aware motion planning in mobile networks, IEEE Trans. Autom. Control 56 (10) (2011) 2478–2485.
[17] J. Le Ny, A. Ribeiro, G.J. Pappas, Adaptive communication-constrained deployment of unmanned vehicle systems, IEEE J. Sel. Areas Commun. 30 (5) (2012) 923–934.
[18] Y. Yan, Y. Mostofi, Co-optimization of communication and motion planning of a robotic operation under resource constraints and in fading environments, IEEE Trans. Wirel. Commun. 12 (4) (2013) 1562–1572.
[19] J. Fink, A. Ribeiro, V. Kumar, Robust control of mobility and communications in autonomous robot teams, IEEE Access 1 (2013) 290–309.
[20] Y. Yan, Y. Mostofi, To go or not to go: on energy-aware and communication-aware robotic operation, IEEE Trans. Control Netw. Syst. 1 (3) (2014) 218–231.
[21] A. Fida, M. Iqbal, T.D. Ngo, Communication-and position-aware reconfigurable route optimization in large-scale mobile sensor networks, EURASIP J. Wirel. Commun. Netw. 207 (2014).
[22] A. Kassir, B. Fitch, S. Sukkarieh, Communication-aware information gathering with dynamic information flow, Int. J. Robot. Res. 34 (2) (2015) 173–200.
[23] Y. Kantaros, M.M. Zavlanos, Communication-aware coverage control for robotic sensor networks, in: 53rd IEEE Conference on Decision and Control, 2014, pp. 6863–6868.
[24] A. Goldsmith, Wireless Communications, Cambridge University Press, New York, 2005.
[25] A. Filippov, Differential Equations with Discontinuous Right-hand Sides, Kluwer Academic Publishers, London, 1988.
[26] F. Clarke, Optimization and Nonsmooth Analysis, Wiley, New York, 1983.
[27] D. Shevitz, B. Paden, Lyapunov stability theory of nonsmooth systems, IEEE Trans. Autom. Control 39 (9) (1994) 1910–1914.
[28] D. Tse, P. Viswanath, Fundamentals of Wireless Communication, Cambridge University Press, New York, 2005.