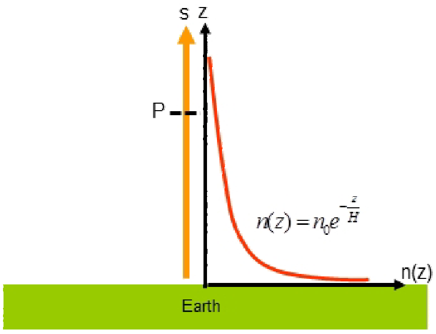
4.28 Radiative transfer: Thermal spectral range: exponential emitter and absorber



a) Now assume that the "exponential atmosphere" is illuminated from below by thermal emission from the Earth's surface and that we are only interested in the thermal infrared spectral range (wavelength $\sim$10 μm), i.e. we can neglect solar radiation and scattering. For simplicity we assume the atmosphere at a constant temperature T_A and the Earth's surface at a temperature T_E. Calculate the radiance $I^{\uparrow}(z)$ upwelling in zenith direction at a certain point P at height z. Since the outgoing radiation is pointing in z direction, use $\tau(z) := \int_0^z \sigma \cdot n(z) \cdot dz$ for optical density here. The Earth's surface may be assumed a black body.

b) What is the upwelling monochromatic radiance $I^{\uparrow}(\infty)$ that an observer would measure at the top-of-the-atmosphere? In case you could not solve part a) of this exercise, assume $I^{\uparrow}(z) = B(T_E)\,\exp\left[-\tau_\infty(1-e^{-\frac{z}{H}})\right] + B(T_A)\,\left[1-\exp\left[-\tau_\infty(1-e^{-\frac{z}{H}})\right]\right]$ with $\tau_\infty = \sigma\, n_0\, H$.

c) Plot the fractional contribution of surface emission and atmospheric emission to the upwelling top-of-the-atmosphere radiance $I^{\uparrow}(\infty)$ as a function of $\tau_\infty = \sigma\, n_0\, H$ for $\sigma \in [10^{-24}, 10^{-20}]$ cm^2 ($T_E = 283\,K, T_A = 255\,K$, $n_0 = 1 \times 10^{16}$ molecule/cm^3, $H = 8$ km). Choose a logarithmic scale for the τ_∞ axis. At what τ_∞ are surface and atmospheric contributions equal?

4.28.1 Solution

First, we define our coordinate system (see Figure 4.25), where the upwelling radiance at z and optical depth $\tilde{\tau}(z)$ is $\tilde{I}^{\uparrow}(z)$.

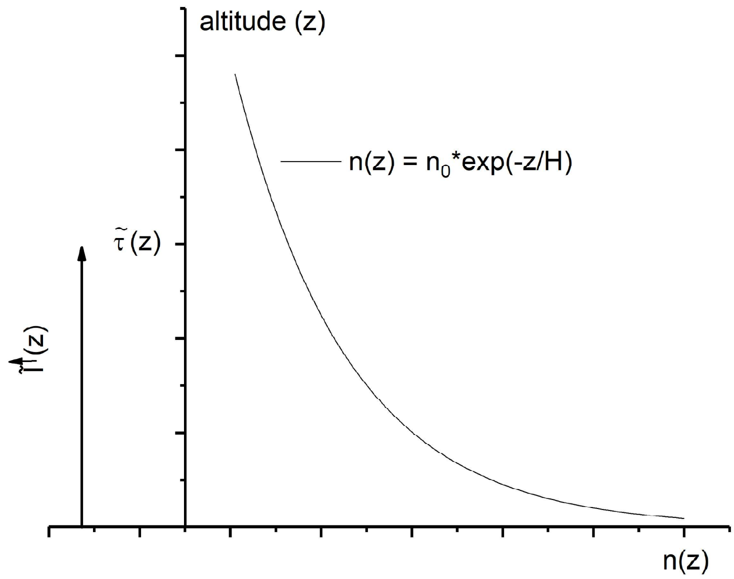


Figure 4.25: Definition of the coordinate system. At the ground $\tilde{I}^{\uparrow}(0)$ is thermally emitted and at any altitude z, or $\tilde{\tau}(z)$ the thermal radiance $\tilde{I}^{\uparrow}(\tilde{\tau}(z))$.

a) At any atmospheric level z the upwelling radiation $\tilde{I}^{\uparrow}(\tilde{\tau}(z))$ has two contributions: (i) the surviving thermal radiation emitted from the ground $B(T_E) \cdot \exp(-\tilde{\tau}(z))$, and (ii) the from 0 to $\tilde{\tau}(z)$ integrated thermal radiation emitted from the atmosphere $\int_0^{\tilde{\tau}(z)} B(T_A(z)) \cdot \exp\left(-(\tilde{\tau}(z) - \tilde{\tau'}(z)\right) \cdot d\tilde{\tau'}(z)$, or in sum

$$\tilde{I}^{\uparrow}(\tilde{\tau}(z)) = B(T_E) \cdot \exp\left(-\tilde{\tau}(z)\right) + \int_0^{\tilde{\tau}(z)} B(T_A(z)) \cdot \exp - \left(\tilde{\tau}(z) - \tilde{\tau'}(z)\right) \cdot d\tilde{\tau'}(z) \quad (4.212)$$

Here for simplicity we assume, that the atmosphere has a uniform temperature, i.e. $B(T_A(z)) = const$. Next we calculate $\tilde{\tau}(z)$ as function of z, H, and n_o

$$\tilde{\tau}(z) = \int_0^z \sigma_a(\lambda) \cdot n(z') \cdot dz' = \int_0^z \sigma_a(\lambda) \cdot n_0 \cdot \exp(-z'/H) \cdot dz' \quad (4.213)$$

or

$$\tilde{\tau}(z) = \tilde{\tau}_\infty \cdot (1 - \exp(-z/H)) \quad (4.214)$$

with $\tau_\infty = \sigma\, n_0\, H$. Inserting $\tilde{\tau}(z)$ into equation 4.212 yields

$$\begin{aligned} \tilde{I}^{\uparrow}(\tilde{\tau}(z)) &= B(T_E) \cdot \exp\left(-\tilde{\tau}_\infty \cdot (1 - \exp(-z/H))\right) \\ &\quad + \left[B(T_A) \cdot \exp\left(-\left(\tilde{\tau}(z) - \tilde{\tau'}(z)\right)\right)\right]_0^{\tilde{\tau}(z)} \end{aligned} \quad (4.215)$$

or

$$\begin{aligned} \tilde{I}^{\uparrow}(\tilde{\tau}_\infty, z, H) &= B(T_E) \cdot \exp\left(-\tilde{\tau}_\infty \cdot (1 - \exp(-z/H))\right) \\ &\quad + B(T_A) \cdot \left(1 - \exp\left(-\tilde{\tau}_\infty \cdot (1 - \exp(-z/H))\right)\right) \end{aligned} \quad (4.216)$$

b) At top of the atmosphere, the term $\exp(-z/H) = 0$, and hence from equation 4.216, we obtain

$$\tilde{I}^{\uparrow}(\tilde{\tau}(z)) = B(T_E) \cdot \exp(-\tilde{\tau}_{\infty}) + B(T_A) \cdot (1 - \exp(-\tilde{\tau}_{\infty})) \tag{4.217}$$

c) Figure 4.26 displays the solution, which indicates that with increasing $\tilde{\tau}_{\infty}$, increasingly less thermal radiation emitted from the ground reaches the space, and an increasing fraction of the total thermal radiation escaping to space originates from the atmosphere.

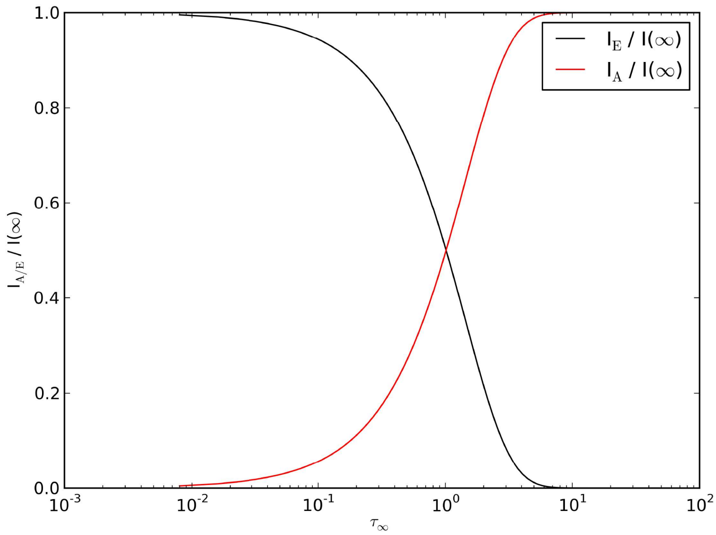


Figure 4.26: Emission of thermal radiation from the ground ($I_E/I(\infty)$) and the atmosphere ($I_A/I(\infty)$) as function of the total atmospheric optical depth τ_{∞}.

Finally, we calculate $\tilde{\tau}(z)$ for which the contribution of the thermal emission from the ground and the atmosphere are the same, i.e., $\tilde{I}_E^{\uparrow}(\tilde{\tau}(z)) = \tilde{I}_A^{\uparrow}(\tilde{\tau}(z))$. From equation 4.217 we finally obtain

$$B(T_E) \cdot \exp(-\tilde{\tau}_{\infty}) = B(T_A) \cdot (1 - \exp(-\tilde{\tau}_{\infty})) \tag{4.218}$$

or

$$\tilde{\tau}_{\infty} = \ln\left(\frac{B(T_E) + B(T_A)}{B(T_A)}\right) \tag{4.219}$$

which for $B(T_A) = B(255K) = 0.57 \cdot B(283K) = 0.57 \cdot B(T_E)$ yields

$$\tilde{\tau}_{\infty} = \ln\left(\frac{1 + 0.57}{0.57}\right) \approx 1 \tag{4.220}$$