Step 1: Measured spectrogram $I_0(p,\tau)$

$$I_0(p,\tau) = \left| \int_{-\infty}^{\infty} E_X(t-\tau) d(p + A(t)) e^{-i\varphi(p,t)} e^{\left(\frac{p^2}{2}+I_p\right)t} dt \right|^2 \tag{1}$$

$$\varphi(p,t) = \int_t^{\infty} \left[pA(t') + \frac{A^2(t')}{2} \right] dt' \tag{2}$$

Experimentally, $I_0(p,\tau)$ is an $N_\epsilon \times N_\tau$ 2-D array with N_ε energy points and N_τ delay points. Usually, the spectrogram is scanned with uniform delay step ($d\tau$). Convert the energy axis to frequency by $f = \varepsilon/2\pi$. The frequency start and end points are f_1 and f_N, respectively. The central frequency is:

$$f_0 = \frac{f_N - f_1}{2} \tag{3}$$

Step 2: Before fed to reconstruction algorithm, the measured spectrogram should be modified to:

$$I_a(p,\tau) = \frac{I_0(p,\tau)}{|d(p)|^2} \tag{4}$$

Why to do it?
With this modification, the spectrogram becomes:

$$I_a(p,\tau) = \left| \int_{-\infty}^{\infty} E_X(t-\tau) \frac{d(p + A(t))}{d(p)} e^{-i\varphi(p,t)} e^{\left(\frac{p^2}{2}+I_p\right)t} dt \right|^2 \tag{5}$$

By applying *central momentum approximation*, p is substituted with p_0 in d and φ, so

$$I_a(p,\tau) \approx \left| \int_{-\infty}^{\infty} E_X(t-\tau) \frac{d(p_0 + A(t))}{d(p_0)} e^{-i\varphi(p_0,t)} e^{\left(\frac{p^2}{2}+I_p\right)t} dt \right|^2 \tag{6}$$

$$\triangleq \left| \int_{-\infty}^{\infty} E_X(t-\tau) G(t) e^{\left(\frac{p^2}{2}+W\right)t} dt \right|^2 \tag{7}$$

here,

$$G(t) = \frac{d(p_0 + A(t))}{d(p_0)} e^{-i\varphi(p_0,t)} \tag{8}$$

Step 3: Interpolating Ia along frequency axis to change the number of frequency points from N_ε to M_ε so that:

1. M_ε is a power of two;
2. the frequency points are uniformly spaced with interval of df:

$$df = \frac{f_N - f_1}{M_\varepsilon}; \tag{9}$$

3. df satisfies:

$$df\, d\tau = L/M_\varepsilon, \tag{10}$$

   here L is an integer.

Why to do it?
After Interpolation, the IAP pulse $P(t)$ is then digitized at equally spaced time points with time step of dt:

$$dt = \frac{1}{df M_\varepsilon} \tag{11}$$

Eq. (11) ensures that the energy points of I_a matches the Fourier transform of P(t). Combining Eq. (10) and Eq. (11), we have:

$$d\tau = L dt, \tag{12}$$

so Eq. (10) requires that the delay interval should be integer multiples of dt.

How to do it?
There may be no solutions for M_ε to fulfill the three conditions simultaneously. However, the interpolation can be done with the following procedure:

1. set L to a proper integer so that:
$$L \geq d\tau(f_N - f_1) \tag{13}$$
2. dt can then be calculated as:
$$dt = \frac{d\tau}{L}; \tag{14}$$
3. set M_ε to be a power of two. Note that M_ε cannot be too small since it determines the time window of the IAP pulse by $T = (M_\varepsilon - 1)dt$:
4. the frequency interval is:
$$df = \frac{1}{M_\varepsilon dt} \tag{15}$$
5. generate the sampling frequency points by keep the central frequency unchanged:
$$f_1 = f_0 - \frac{M_\varepsilon}{2} df, \tag{16}$$
$$f_N = f_0 + \left(\frac{M_\varepsilon}{2} - 1\right) df, \tag{17}$$
$$\tag{18}$$
so,
$$f_i = f_0 + \left(i - 1 - \frac{M_\varepsilon}{2}\right) df \qquad (i = 1, 2, ..., M_\varepsilon) \tag{19}$$
6.