

1 The Backmap Python Module: How a 2 Simpler Ramachandran Number Can 3 Simplify the Life of a Protein Simulator

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6 ABSTRACT

7 Protein backbones display complicated structures that often undergo numerous types of structural
8 transformations. Due to the large number of structural degrees of freedom available to a backbone,
9 it is often difficult to assess exactly where and how regions of a protein structure undergo structural
10 transformation. This large structural phase makes it hard to survey new structural data, such as molecular
11 dynamics trajectories or NMR-derived structural ensembles. This report discusses the Ramachandran
12 number \mathcal{R} as a residue-level structural metric that could simplify the life of anyone contending with large
13 numbers of structural data associated with protein backbones. In particular, this report 1) discusses a
14 much simpler closed form of \mathcal{R} that makes it more easy to calculate, 2) shows how \mathcal{R} dramatically reduces
15 the dimensionality of the protein backbone, thereby making it ideal for simultaneously interrogating large
16 number of protein structures. In short, \mathcal{R} is a simple and succinct descriptor of protein backbones and
17 their dynamics.

18 INTRODUCTION

19 Proteins are a class of biomolecules unparalleled in their functionality (Berg *et al.*, 2010). A natural
20 protein may be thought of as a linear chain of amino acids, each normally sourced from a repertoire of 20
21 naturally occurring amino acids. Proteins are important partially because of the structures that they access:
22 the conformations (conformational ensemble) that a protein assumes determines the functions available
23 to that protein. However, all proteins are dynamic: even stable proteins undergo long-range motions
24 in its equilibrium state; i.e., they have substantial diversity in their conformational ensemble (Mannige,
25 2014). Additionally, a number of proteins undergo conformational transitions, without which they may
26 not properly function. Finally, some proteins – intrinsically disordered proteins – display massive disorder
27 whose conformations dramatically change over time (Uversky, 2003; Fink, 2005; Midic *et al.*, 2009;
28 Espinoza-Fonseca, 2009; Uversky and Dunker, 2010; Tompa, 2011; Sibille and Bernado, 2012; Kosol
29 *et al.*, 2013; Dunker *et al.*, 2013; Geist *et al.*, 2013; Baruah *et al.*, 2015), and whose characteristic
30 structures are still not well-understood (Beck *et al.*, 2008).

31 Large-scale changes in a protein occur due to changes in protein backbone conformations. Fig. 1 is a
32 cartoon representation of a peptide/protein backbone, with the backbone bonds themselves represented
33 by darkly shaded bonds. Ramachandran *et al.* (1963) had recognized that the backbone conformational

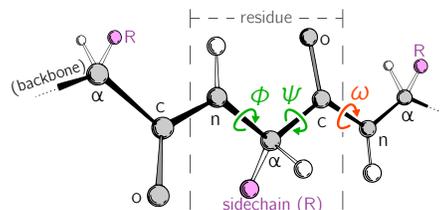


Figure 1. Backbone conformational degrees of freedom dominantly depend on the dihedral angles ϕ and ψ (green), and to a smaller degree depend on the third dihedral angle (ω ; red) as well as bond lengths and angles (unmarked).

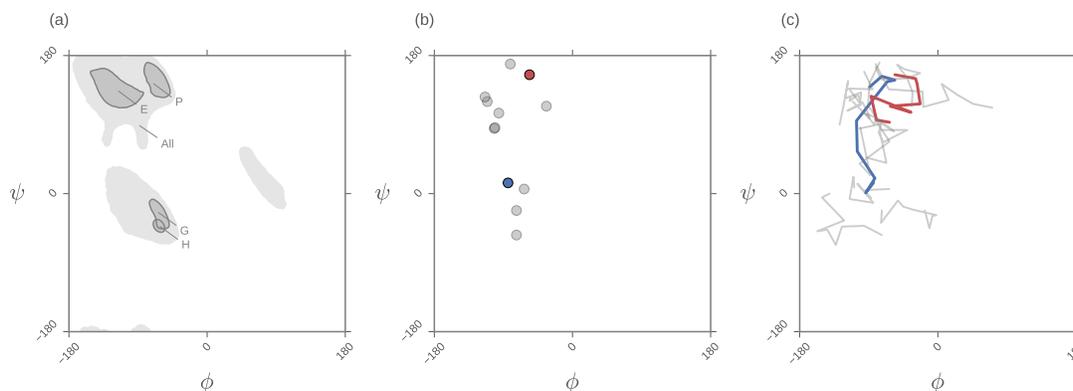


Figure 2. While the Ramachandran plot is useful for getting a *qualitative* sense of peptide backbone structure (a, c), it is not a convenient representation for exploring peptide backbone dynamics (c).

34 degrees of freedom available to an amino acid (residue) i is almost completely described by only two
 35 dihedral angles: ϕ_i and ψ_i (Fig. 1, green arrows). Today, protein structures described in context of the
 36 two-dimensional (ϕ, ψ) -space are called Ramachandran plots.

37 The Ramachandran plot is recognized as a powerful tool for two reasons: 1) it serves as a map
 38 for structural ‘correctness’ (Laskowski *et al.*, 1993; Hoofst *et al.*, 1997; Laskowski, 2003), since many
 39 regions within the Ramachandran plot space are energetically not permitted (Momen *et al.*, 2017); and
 40 2) it provides a qualitative snapshot of the structure of a protein (Berg *et al.*, 2010; Alberts *et al.*, 2002;
 41 Subramanian, 2001). For example, particular regions within the Ramachandran plot indicate the presence
 42 of particular secondary locally-ordered structures such as the α -helix and β -sheet (see Fig. 2a).

43 While the Ramachandran plot has been useful as a measure of protein backbone conformation, it is
 44 not popularly used to assess structural dynamism and transitions (unless specific knowledge exists about
 45 whether a particular residue is believed to undergo a particular structural transition). This is because
 46 of the two-dimensionality of the plot: describing the behavior of every residue involves tracking its
 47 position in two-dimensional (ϕ, ψ) space. For example, a naive description of positions of a peptide in a
 48 Ramachandran plot (Fig. 2b) needs more annotations for a per-residue analysis of the peptide backbone’s
 49 structure. Given enough residues, it would be impractical to track the position of each residue within a
 50 plot. This is compounded with time, as each point in (b) becomes a curve (c), further confounding the
 51 situation. The possibility of picking out previously unseen conformational transitions and dynamism
 52 becomes a logistical impracticality. As indicated above, this impracticality arises primarily from the fact
 53 that the Ramachandran plot is a two-dimensional map.

54 Consequently, there has been no single compact descriptor of protein structure. This impedes that
 55 naïve or hypothesis-free exploration of new trajectories/ensembles. For example, tracking changes in
 56 protein trajectory is either overly detailed or overly holistic: an example of an overly detailed study is the
 57 tracking on exactly one or a few atoms over time (this already poses a problem, since we would need to
 58 know exactly which atoms are expected to partake in a transition); an example of a holistic metric is the
 59 radius of gyration (this also poses a problem, since we will never know which residues contribute to a
 60 change in radius of gyration without additional interroogation). With protein dynamics undergoing a new
 61 renaissance – especially due to intrinsically disordered proteins and allostery – having hypothesis-agnostic
 62 yet detailed (residue-level) metrics of protein structure has become even more relevant.

63 It has recently been shown that the two Ramachandran backbone parameters (ϕ, ψ) may be conve-
 64 niently combined into a single number – the Ramachandran *number* [$\mathcal{R}(\phi, \psi)$ or simply \mathcal{R}] – with little
 65 loss of information (Mannige *et al.*, 2016). In a previous report, detailed discussions were provided
 66 regarding the reasons behind and derivation of \mathcal{R} (Mannige *et al.*, 2016). This report provides a simpler
 67 version of the equation previously published (Mannige *et al.*, 2016), and further discusses how \mathcal{R} may be
 68 used to provide information about protein ensembles and trajectories. Finally, we introduce a software
 69 package – BACKMAP – that can be used by to produce MAPs that describe the behavior of a protein
 70 backbone within user-inputted conformations, structural ensembles and trajectories. This package is
 71 presently available on GitHub (<https://github.com/ranjanmannige/BackMap>).

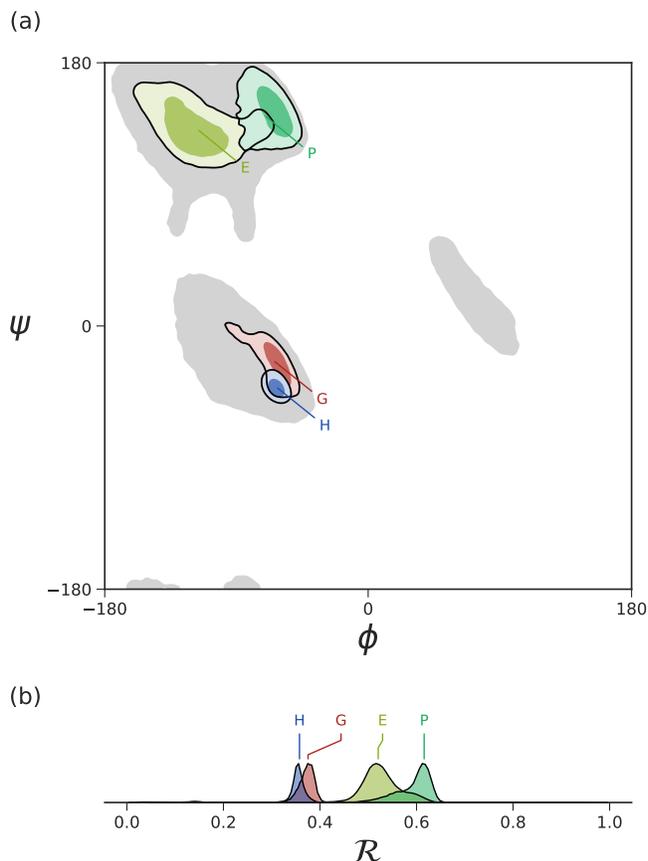


Figure 3. The distribution of dominant regular secondary structures are shown in $[\phi, \psi]$ -space (a) and in \mathcal{R} -space (b). Both Ramachandran plots (a) and Ramachandran ‘lines’ (b) show equivalent resolution of secondary structure, allowing for a more compact representation of Ramachandran plots [Mannige *et al.* \(2016\)](#).

72 INTRODUCING THE *SIMPLIFIED* RAMACHANDRAN NUMBER (\mathcal{R})

73 The Ramachandran number is both an idea and an equation. Conceptually, the Ramachandran number (\mathcal{R})
 74 is any closed form that collapses the dihedral angles ϕ and ψ into one structurally meaningful number
 75 ([Mannige *et al.*, 2016](#)). [Mannige *et al.* \(2016\)](#) presented a version of the Ramachandran number that
 76 was complicated in closed form, thereby reducing its utility. Here, a much more simplified version of the
 77 Ramachandran number is introduced. Section 1.1 shows how this simplified form was derived from the
 78 original closed form (Eqns. 3 and 4).

Given arbitrary limits of $\phi \in [\phi_{\min}, \phi_{\max}]$ and $\psi \in [\psi_{\min}, \psi_{\max}]$, where the minimum and maximum values differ by 360° , the most general and accurate equation for the Ramachandran number is

$$\mathcal{R}(\phi, \psi) \equiv \frac{\phi + \psi - (\phi_{\min} + \psi_{\min})}{(\phi_{\max} + \psi_{\max}) - (\phi_{\min} + \psi_{\min})}. \quad (1)$$

For consistency, we maintain throughout this paper that $\phi_{\min} = \psi_{\min} = -180^\circ$ or $-\pi$ radians, which makes

$$\mathcal{R}(\phi, \psi) = \frac{\phi + \psi + 2\pi}{4\pi}. \quad (2)$$

79 As evident in Fig. 3, the distributions within the Ramachandran plot are faithfully reflected in corre-
 80 sponding distributions within Ramachandran number space. This paper shows how the Ramachandran
 81 number is both compact enough and informative enough to generate immediately useful graphs (map) of
 82 a dynamic protein backbone.

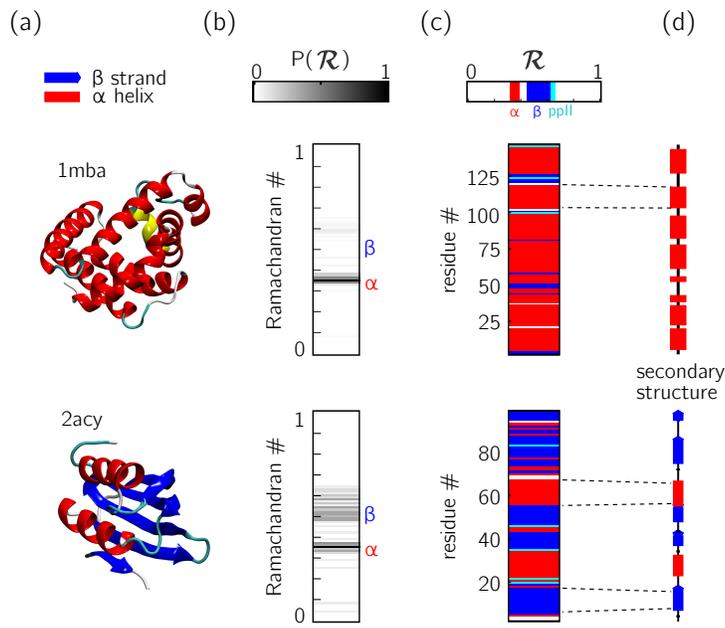


Figure 4. Two types of \mathcal{R} -codes. Digesting protein structures (a) using \mathcal{R} numbers either as histograms (b) or per-residue codes (c) allow for compact representations of salient structural features. For example, a single glance at the histograms indicate that protein 1mba is likely all α -helical, while 2acy is likely a mix of α -helices and β -sheets. Additionally, residue-specific codes (c) not only indicate secondary structure content, but also exact secondary structure stretches (compare to d), which gives a more complete picture of how the protein is linearly arranged.

83 REASON TO USE THE RAMACHANDRAN NUMBER

84 Ramachandran numbers are more compact than one might realize

85 An important aspect of the Ramachandran number (\mathcal{R}) lies in its compactness compared to the traditional
 86 Ramachandran pair (ϕ, ψ) . Say we have an N -residue peptide. Then, switching from (ϕ, ψ) to \mathcal{R} appears
 87 to only reduce the number of variables from $2N$ to N , and hence by half. However, (ϕ, ψ) values are
 88 *coupled*, i.e., for any N -length peptide, any ordering of $[\phi_1, \phi_2, \dots, \phi_N, \psi_1, \psi_2, \dots, \psi_N]$ can not describe
 89 the structure, it is only *pairs* – $[(\phi_1, \psi_1), (\phi_2, \psi_2), \dots, (\phi_N, \psi_N)]$ – that can. Therefore, we must think of
 90 switching from (ϕ, ψ) -space to \mathcal{R} -space as a switch in structure space per residue from N two-tuples
 91 (ϕ_i, ψ_i) that reside in $\phi \times \psi$ space to N single-dimensional numbers (\mathcal{R}_i).

92 The value of this conversion is that the structure of a protein can be described in various one-
 93 dimensional arrays (per-structure “Ramachandran codes” or “ \mathcal{R} -codes”), which, when arranged vertical-
 94 ly/columnarly, constitute easy to digest codes. See, e.g., Fig. 4.

95 Ramachandran codes are stackable

96 In addition to assuming a small form factor, \mathcal{R} -codes may then be *stacked* side-by-side for visual and
 97 computational analysis. There lies its true power.

98 For example, the one- \mathcal{R} -to-one-residue mapping means that the entire residue-by-residue structure
 99 of a protein can be shown using a string of \mathcal{R}_i s (which would show regions of secondary structure and
 100 disorder, for starters). Additionally, an entire protein’s backbone makeup can be shown as a histogram in
 101 \mathcal{R} -space (which may reveal a protein’s topology). The power of this format lies not only in the capacity
 102 to distill complex structure into compact spaces, but in its capacity to display *many* complex structures in
 103 this format, side-by-side (stacking).

104 Peptoid nanosheets (Mannige *et al.*, 2015) will be used here as an example of how multiple structures,
 105 in the form of \mathcal{R} -codes, may be stacked to provide immediately useful pictograms. Peptoid nanosheets
 106 are a recently discovered peptide-mimic that were shown to display a novel secondary structure (Mannige
 107 *et al.*, 2015). In particular, each peptoid within the nanosheet displays backbone conformations that
 108 alternate in chirality, causing the backbone to look like a meandering snake that nonetheless maintains an

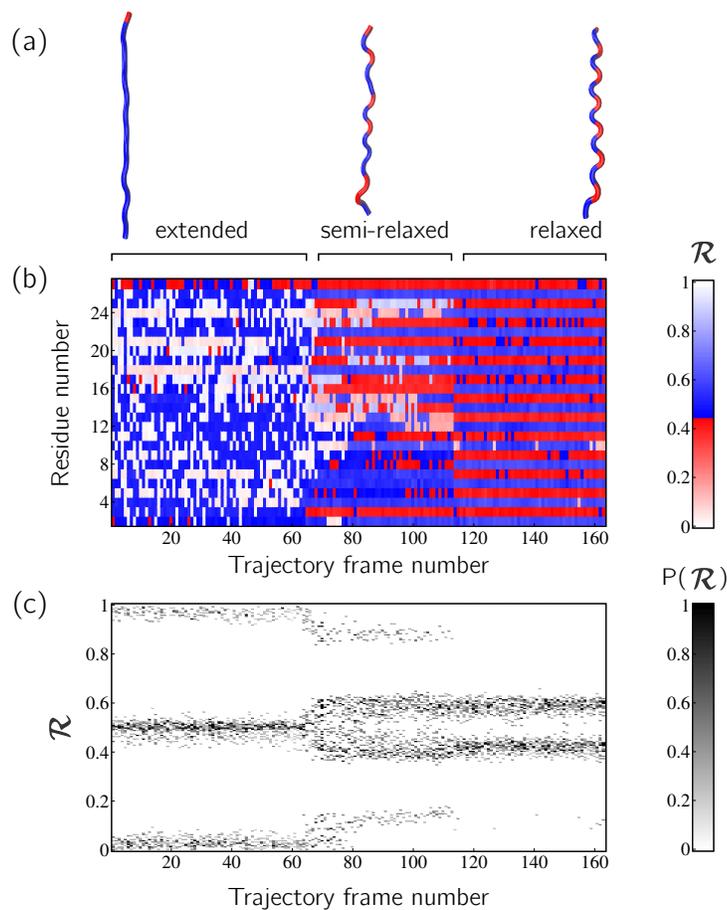


Figure 5. Stacked \mathcal{R} -codes provide useful information at a glance.

109 overall linear direction. This secondary structure was discovered by first setting up a nanosheet where all
 110 peptoid backbones are restrained in the extended format (Fig. 5a, left), after which the restraints were
 111 energetically softened (a, middle) and completely released (a, right). As evident in Fig. 5b and Fig. 5c, the
 112 two types of \mathcal{R} -code stacks display salient information at first glance: 1) Fig. 5b shows that the extended
 113 backbone first undergoes some rearrangement with softer restraints, and then becomes much more binary
 114 in arrangement as we look down the backbone (excepting the low-order region in the middle, unshown in
 115 Fig. 5a); and 2) Fig. 5c shows that lifting restraints on the backbone causes a dramatic change in backbone
 116 topology, namely a birth of a bimodal distribution evident in the two parallel bands.

117 By utilizing \mathcal{R} , maps such as those in Fig. 5 provide information about every ϕ and ψ within the
 118 backbone. As such, these maps are dubbed MAPs, for Multi Angle Pictures. A Python package called
 119 BACKMAP created Fig. 5a and b, which is provided as a GitHub repository at [https://github.com/
 120 ranjanmannige/BackMap](https://github.com/ranjanmannige/BackMap). BACKMAP takes in a PDB structure file containing a single structure, or
 121 multiple structures separated by the code ‘MODEL’.

122 **Other uses for \mathcal{R} : picking out subtle differences from high volume of data**

123 This section expands on the notion that \mathcal{R} -numbers – due to their compactness/stackability – can be used
 124 to pick out backbone structural trends that would be hard to decipher using any other metric. For example,
 125 it is well known that prolines (P) display unusual backbone behavior: in particular, proline backbones
 126 occupy structures that are close to but distinct from α -helical regions. Due to the two-dimensionality
 127 of Ramachandran plots (Fig. 6a), such distinctions are hard to visually pick out from Ramachandran
 128 plots. However, stacking per-amino-acid \mathcal{R} -codes side by side make such differences patent (Fig. 6b; see
 129 arrow).

130 It is also known that amino acids preceding prolines display unusual shift in chirality. For example,
 131 Fig. 7 shows that amino acids appearing before prolines and glycines behave much more differently than

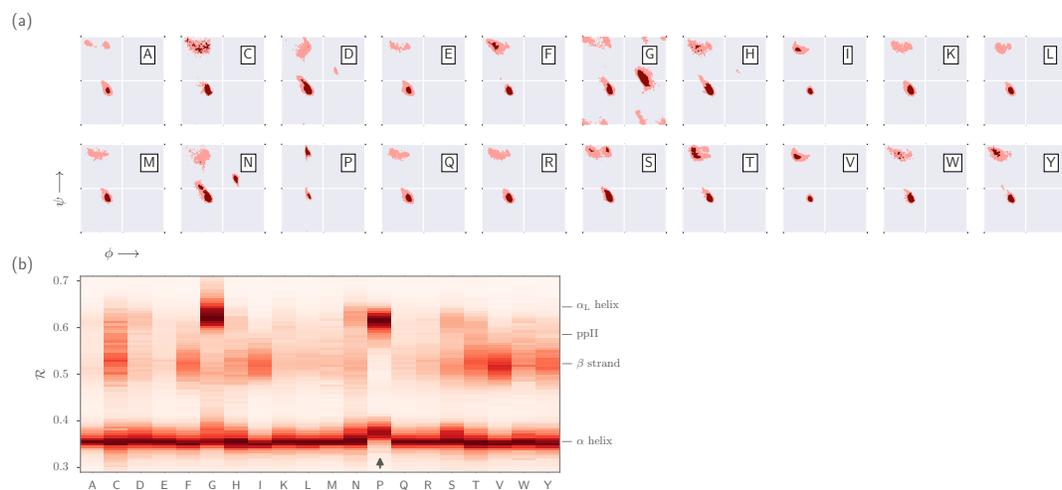


Figure 6. Ramachandran lines are stackable – Part I. Panel (a) shows the per-amino acid backbone behavior of an average protein found in the protein databank (PDB). While these plots are useful, it is difficult to compare such plots. For example, it is hard to pick out the change in the α -helical region of the proline plot (P). However, when we convert Ramachandran plots to Ramachandran *lines* [by converting $(\phi_i, \psi_i) \rightarrow \mathcal{R}_i$], we are able to conveniently “stack” Ramachandran lines calculated for each residue. Then, even visually, it is obvious that proline does not occupy the canonical α -helix region, which is not evident to an untrained eye in (a).

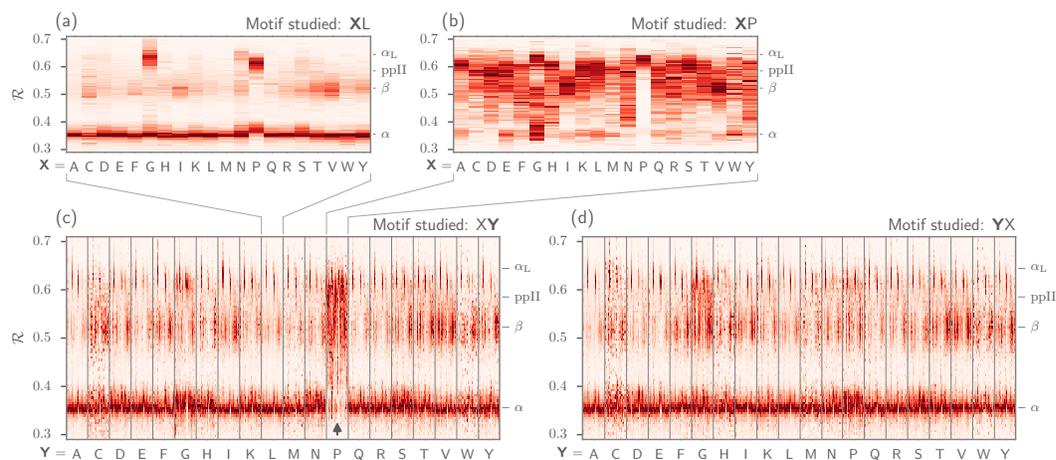


Figure 7. Ramachandran lines are stackable – Part II. Similar to Fig. 6b, Panel (a) represents the behavior of an amino acid ‘Y’ situated *before* a leucine (assuming that we are reading a sequence from the N terminal to the C terminal). Panel (b) similarly represents the behavior of specific amino acids situated before a proline. While residues preceding a leucine behave similarly to their average behavior (Fig. 6a), most residues preceding prolines appear to be enriched in structures that change ‘direction’ or backbone chirality ($\mathcal{R} > 0.5$). Panel (c) shows the behavior of individual amino acids when situated before each of the 20 amino acids. This graph shows a major benefit of side-by-side Ramachandran line “stacking”: general trends become much more obvious. For example, it is evident that glycines and prolines dramatically modify the structure of an amino acid preceding it (compared to average behavior of amino acids in Fig. 6b). This trend is not as strong when considering amino acids that *follow* glycines or prolines (c). Such trends, while previously discovered [e.g., [Gunasekaran et al. \(1998\)](#); [Ho and Brasseur \(2005\)](#)], would not be accessible when naïvely considering Ramachandran plots because one would require 400 (20×20) distinct Ramachandran plots to compare.

132 they would otherwise. While these results have been discussed previously (Gunasekaran *et al.*, 1998; Ho
133 and Brasseur, 2005), they were reported more than 30 years after the first structures were published; they
134 would have been relatively easy to find if \mathcal{R} -codes were to be used regularly.

135 The relationships in Figs. 6 and 7 show how subtle changes in structure can be easily picked out when
136 structures are stacked side-by-side in the form of \mathcal{R} -codes. Such subtle changes are often witnessed when
137 protein backbones transition from one state to another.

138 USING THE BACKMAP PYTHON MODULE

139 Installation

140 BACKMAP may either be downloaded from the github repository, or installed directly by running the
141 following line in the command prompt (assuming that pip exists): `> pip install backmap`

142 First simple test

143 The simplest test would be to generate Ramachandran numbers from (ϕ, ψ) pairs:

```
144 

---

145 # Import module 1  
146 import backmap 2  
147 # Convert (phi, psi) to R 3  
148 print backmap.R(phi=0,psi=0) # Expected output: 0.5 4  
149 print backmap.R(-180, -180) # Expected output: 0.0 5  
150 print backmap.R(180, 180) # Expected output: 1.0 (equivalent in meaning to 0) 6  


---


```

152 Basic usage for creating Multi-Angle Pictures (MAPs)

153 As seen above, the generation of Ramachandran numbers from (ϕ, ψ) pairs is simple. However, creating
154 MAPs – Multi-Angle Pictures of protein backbones – requires a few more steps (present as a test in the
155 downloadable module):

156 1. Select and read a protein PDB structure

157 Each trajectory frame must be a set of legitimate protein databank "ATOM" records separated by
158 "MODEL" keywords.

```
159 

---

160 import backmap 1  
161 pdbfn = './pdbs/nanosheet_birth_U7.pdb' # Set pdb name 2  
162 data = backmap.read_pdb(pdbfn) # READ PDB in the form of a matrix with columns 3  


---


```

164 Here, 'data' is a 2d array with four columns ['model', 'chain', 'resid', 'R']. The first row of
165 'data' is the header (i.e., the name of the column, e.g., 'model'), with values that follow.

166 2. Select color scheme (color map)

167 In addition to custom colormaps listed in the next section, one can also use standardly available at
168 matplotlib.org (e.g., 'Reds' or 'Reds_r').

```
169 

---

170 # setting the name of the colormap 4  
171 cmap = "SecondaryStructure" 5  


---


```

173 3. Draw per-chain MAPs

```
174 

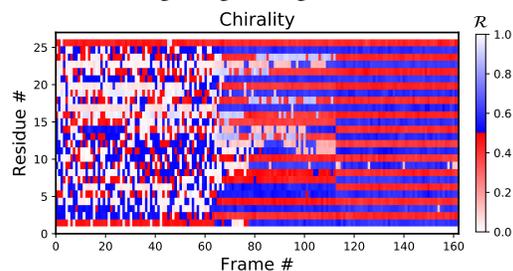
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175 # Grouping by chain 6  
176 grouped_data = backmap.group_by(data, group_by='chain', 7  
177                                columns_to_return=['model', 'resid', 'R']) 8  
178 for chain in grouped_data.keys(): # Going through each chain 9  
179     # Getting the X,Y,Z values for each entry 10  
180     models, residues, Rs = grouped_data[chain] 11  
181     # Finally, creating (but not showing) the graph 12  
182     backmap.draw_xyz(X = models, Y = residues, Z = Rs 13  
183                    , xlabel='Frame #', ylabel="Residue #", zlabel='$\mathcal{R}$' 14  
184                    , cmap = cmap, title = "Chain: '"+chain+"'" 15  
185                    , vmin=0, vmax=1) 16  
186     # Now, we display the graph: 17  
187     plt.show() # ... one can also use plt.savefig() to save to file 18  


---


```

189 As one would expect, this is the business end of the code. By changing how one assigns values
 190 to 'X' and 'Y', one can easily construct and draw other types of graphs such as time-resolved
 191 histograms, root mean squared fluctuations, root mean squared deviation, etc. Running the module
 192 as a standalone script would produce all these graphs automatically. 'plt.show()' would result
 193 in the following image being rendered:



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195 Creating custom graphs

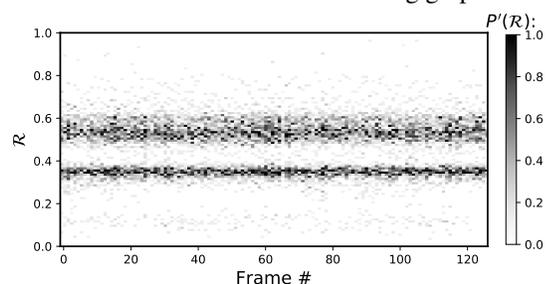
196 Other types of graphs can be easily created by modifying part three of the code above. For example, the
 197 following code creates histograms of R, one for each model (starting from line 10 above).

```

198 for chain in grouped_data.keys():
199     models, residues, Rs = grouped_data[chain]
200
201     'Begin custom code'
202     X = []; Y=[]; Z=[]; # Will set X=model, Y=R, Z=P(R)
203     # Bundling the three lists into one 2d array
204     new_data = np.array(zip(models, residues, Rs))
205     # Getting all R values, model by model
206     for m in sorted(set(new_data[:,0])): # column 0 is the model column
207         # Getting all Rs for that model #
208         current_rs = new_data[np.where(new_data[:,0]==m)][:,2] # column 2 contains R
209         # Getting the histogram
210         a,b = np.histogram(current_rs, bins=np.arange(0,1.01,0.01))
211         max_count = float(np.max(a))
212         for i in range(len(a)):
213             X.append(m); Y.append((b[i]+b[i+1])/2.0); Z.append(a[i]/float(np.sum(a)));
214         'End custom code'
215
216     # Finally, creating (but not showing) the graph
217     draw_xyz(X = X, Y = Y, Z = Z
218             , xlabel = 'Frame #', ylabel = "\mathcal{R}", zlabel = "$P'(\mathcal{R})$"
219             , cmap = 'Greys', ylim=[0,1])
220     plt.yticks(np.arange(0,1.00001,0.2))
221     # Now, we display the graph:
222     plt.show() # ... one can also use plt.savefig() to save to file
223
  
```

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225 The code above results in the following graph:



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227 Available color schemes (CMAPs)

228 Aside from the general color maps (cmaps) that exist in matplotlib (e.g., 'Greys', 'Reds', or, god forbid,
 229 'jet'), BACKMAP provides two new colormaps: 'Chirality', 'SecondaryStructure'. Fig. 8
 230 shows how a single protein ensemble may be described using these schematics. As illustrated in Fig. 8b,

231 cmapns available within the standard matplotlib package do not distinguish between major secondary
 232 structures well, to a great extent, while those provided by BACKMAP do.

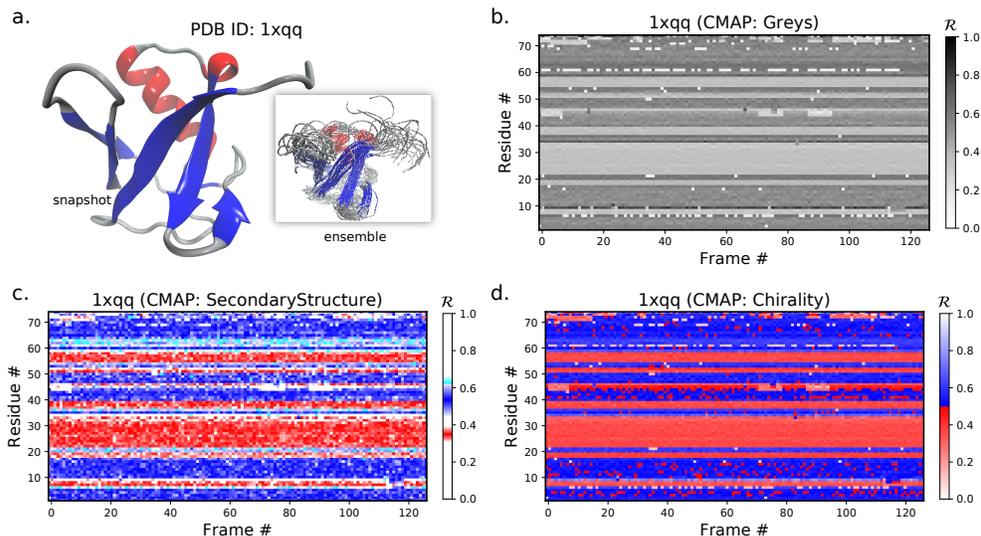


Figure 8. A protein ensemble (a) along with some MAPs colored with different themes (b-d). Panels (c) and (d) are provided by the BACKMAP module. In Panel (a), β -sheets are shown in blue and all helices are shown in red.

232

233 CONCLUSION

234 A simpler Ramachandran number is reported – $\mathcal{R} = (\phi + \psi + 2\pi)/(4\pi)$ – which, while a single number,
 235 provides much information. For example, as discussed in [Mannige *et al.* \(2016\)](#), \mathcal{R} values above 0.5 are
 236 left-handed, while those below 0.5 are right handed, \mathcal{R} values close to 0, 0.5 and 1 are extended, β -sheets
 237 occupy \mathcal{R} values at around 0.52, right-handed α -helices hover around 0.34. Given the Ramachandran
 238 number’s ‘stackability’, single graphs can hold a detailed information of the progression/evolution of
 239 molecular trajectories. Indeed, [Fig. 7](#) shows how 400 distinct Ramachandran plots can easily be fit into
 240 one graph when using \mathcal{R} . Finally, a python script/module (BACKMAP) has been provided in an online
 241 [GitHub repository](#).

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248 1 APPENDIX

249 1.1 Simplifying the Ramachandran number (\mathcal{R})

250 This section will derive the simplified Ramachandran number presented in this paper from the more
 251 complicated looking Ramachandran number introduced previously [Mannige *et al.* \(2016\)](#).

Assuming the bounds $\phi, \psi \in [-180^\circ, 180^\circ)$, and the range λ equals 360° , the previously described
 Ramachandran number takes the form

$$\mathcal{R}(\phi, \psi) \equiv \frac{R_{\mathbb{Z}}(\phi, \psi) - R_{\mathbb{Z}}(\phi_{\min}, \phi_{\min})}{R_{\mathbb{Z}}(\phi_{\max}, \phi_{\max}) - R_{\mathbb{Z}}(\phi_{\min}, \phi_{\min})}, \quad (3)$$

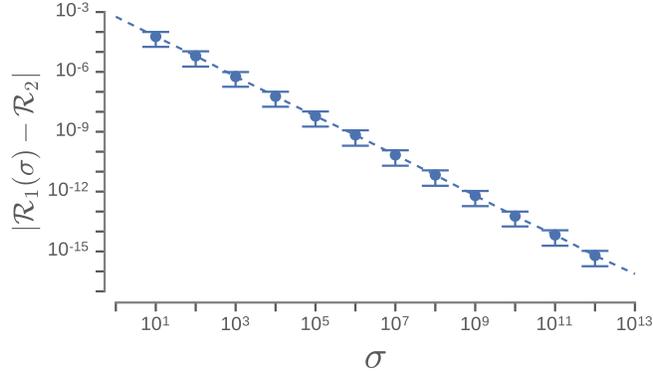


Figure 9. The increase in the accuracy measure (σ) for the original Ramachandran number (Eqn. 4) results in values that tend towards the new Ramachandran number proposed in this paper (Eqn. 2).

where, $\mathcal{R}(\phi, \psi)$ is the Ramachandran number with range $[0, 1)$, and $R_{\mathbb{Z}}(\phi, \psi)$ is the *unnormalized* integer-spaced Ramachandran number whose closed form is

$$R_{\mathbb{Z}}(\phi, \psi) = \left\lfloor (\phi - \psi + \lambda)\sigma/\sqrt{2} \right\rfloor + \left\lfloor \sqrt{2}\lambda\sigma \right\rfloor \left\lfloor (\phi + \psi + \lambda)\sigma/\sqrt{2} \right\rfloor. \quad (4)$$

Here, $\lfloor x \rfloor$ rounds x to the closest integer value, σ is a scaling factor, discussed below, and λ is the range of an angle in degrees (i.e., $\lambda = \phi_{\max} - \phi_{\min}$). Effectively, this equation does the following. **1**) It divides up the Ramachandran plot into $(360^\circ\sigma^{1/\circ})^2$ squares, where σ is a user-selected scaling factor that is measured in reciprocal degrees [see Fig. 8b in Mannige *et al.* (2016)]. **2**) It then assigns integer values to each square by setting the lowest integer value to the bottom left of the Ramachandran plot ($\phi = -180^\circ, \psi = -180^\circ$; green arrow in Fig. 1b) and proceeding from the bottom left to the top right by iteratively slicing down $-1/2$ sloped lines and assigning increasing integer values to each square that one encounters. **3**) Finally, the equation assigns any (ϕ, ψ) pair within $\phi, \psi \in [-\phi_{\min}, \phi_{\max})$ to the integer value ($R_{\mathbb{Z}}$) assigned to the divided-up square that they exist in.

However useful Eqn. 3 is, the complexity of the equation may be a deterrent towards utilizing it. This paper reports a simpler equation that is derived by taking the limit of Eqn. 3 as σ tends towards ∞ . In particular, when $\sigma \rightarrow \infty$, Eqn. 3 becomes

$$\mathcal{R}(\phi, \psi) = \lim_{\sigma \rightarrow \infty} \mathcal{R}(\phi, \psi) = \frac{\phi + \psi + \lambda}{2\lambda} = \frac{\phi + \psi + 2\pi}{4\pi}. \quad (5)$$

Conformation of this limit is shown numerically in Fig. 9. Since larger σ s indicate higher accuracy, $\lim_{\sigma \rightarrow \infty} \mathcal{R}(\phi, \psi)$ represents an exact representation of the Ramachandran number. Using this closed form, this report shows how both static structural features and complex structural transitions may be identified with the help of Ramachandran number-derived plots.

1.2 Other frames of reference

The Ramachandran number shown in Eqn. 5 expects $\phi, \psi \in [-\lambda/2, \lambda/2)$. Given arbitrary limits of $\phi \in [\phi_{\max}, \phi_{\min})$ and $\psi \in [\psi_{\max}, \psi_{\min})$, the most general equation for the Ramachandran number is

$$\mathcal{R}(\phi, \psi) \equiv \frac{\phi + \psi - (\psi_{\min} + \psi_{\min})}{(\psi_{\max} + \psi_{\max}) - (\psi_{\min} + \psi_{\min})}. \quad (6)$$

For example, assuming that $\phi, \psi \in [0, 2\pi)$, the Ramachandran number in that frame of reference will be

$$\mathcal{R}(\phi, \psi)_{\phi, \psi \in [0, 2\pi)} = \frac{\phi + \psi}{4\pi}. \quad (7)$$

However, in doing so, the meaning of the Ramachandran number will change. The rest of this manuscript will always assume that all angles range between $-\pi$ (-180°) and π (180°)

268 2 A SIGNED RAMACHANDRAN NUMBER

An additional Ramachandran number – the *signed* Ramachandran number \mathcal{R}_S – is introduced here for backbones that are achiral. \mathcal{R}_S is identical to the original number in magnitude, but which changes sign from + to – as you approach \mathcal{R} numbers that are to the right (or below) the positively sloped diagonal. I.e.,

$$\mathcal{R}_S = \begin{cases} \mathcal{R} & , \text{ if } \psi \geq \phi \\ \mathcal{R} \times -1 & , \text{ if } \psi < \phi \end{cases} \quad (8)$$

269 This metric is important for those glycine-rich peptides (and peptide-mimics such as peptoids) that both
270 left and right regions of the Ramachandran plot; this is because, for such backbones, each $-1/2$ -sloping
271 slide of the Ramachandran plot may intersect more than one relevant region of the Ramachandran plot,
272 which would put two structurally disparate regions within the Ramachandran plot close in \mathcal{R} -space. The
273 signed Ramachandran plot \mathcal{R}_S minimizes the probability of this happening. However, very few residues
274 within structural databases occupy the right side of the Ramachandran plot (3.5%), which means that
275 signed Ramachandran plots would only be useful in special cases (and possibly for IDPs). For this reason,
276 we will proceed below with a focus on the more relevant Ramachandran number \mathcal{R} .

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