

性能测试

Image	ours	SimpleTex	阿里达摩院	pix2text	新东方
$\prod_{j=1}^n G(i,j)^{1/n} = \prod_{k=1}^n \prod_{j=1}^n x_k^{a_k(i,j)/n} = \prod_{k=1}^i x_k^{1/i} = \sqrt[i]{x_1 x_2 \cdots x_i}.$	$\prod_{j=1}^n G(i,j)^{1/n} = \prod_{k=1}^n \prod_{j=1}^n x_k^{a_k(i,j)/n} = \prod_{k=1}^i x_k^{1/i} = \sqrt[i]{x_1 x_2 \cdots x_i}.$	$\prod_{j=1}^n G(i,j)^{1/n} = \prod_{k=1}^n \prod_{j=1}^n x_k^{a_k(i,j)/n} = \prod_{k=1}^i x_k^{1/i} = \sqrt[i]{x_1 x_2 \cdots x_i}.$	$\prod_{j=1}^n G(\mathrm{i},\mathrm{j})^{1/\mathrm{n}} = \prod_{\mathrm{k}=1}^{\mathrm{n}} \prod_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{x}_{\mathrm{k}}^{\mathrm{a}_{\mathrm{k}}(\mathrm{i},\mathrm{j})/\mathrm{n}} = \prod_{\mathrm{k}=1}^{\mathrm{i}} \mathrm{x}_{\mathrm{k}}^{1/\mathrm{i}} = \sqrt[\mathrm{i}]{\mathrm{x}_1 \mathrm{x}_2 \cdots \mathrm{x}_{\mathrm{i}}}.$	$\prod_{j=1}^n G(i,j)^{1/n} = \prod_{k=1}^n \prod_{j=1}^n x_k^{a_k(i,j)/n} = \prod_{k=1}^i x_k^{1/i} = \sqrt[i]{x_1 x_2 \cdots x_i}.$	$/n=\prod_{k=1}^n x_k a_k(ij) / n = \prod_{k=1}^i x_k^n / i = \sqrt[n]{x_1 x_2 \cdots x_i}.$
$\frac{1}{n}\sum_{j=1}^n H(i,j)^{-1} = \frac{1}{n}\sum_{k=1}^n \sum_{j=1}^n a_k(i,j)x_k^{-1} = \frac{1}{n}\sum_{k=1}^n x_k^{-1}\sum_{j=1}^n a_k(i,j) = \frac{1}{i}\sum_{k=1}^i x_k^{-1}$	$\frac{1}{n}\sum_{j=1}^n H(i,j)^{-1} = \frac{1}{n}\sum_{k=1}^n \sum_{j=1}^n a_k(i,j)x_k^{-1} = \frac{1}{n}\sum_{k=1}^n x_k^{-1}\sum_{j=1}^n a_k(i,j) = \frac{1}{i}\sum_{k=1}^i x_k^{-1}$	$\frac{1}{n}\sum_{j=1}^n H(i,j)^{-1} = \frac{1}{n}\sum_{k=1}^n \sum_{j=1}^n a_k(i,j)x_k^{-1} = \frac{1}{n}\sum_{k=1}^n x_k^{-1}\sum_{j=1}^n a_k(i,j) = \frac{1}{i}\sum_{k=1}^i x_k^{-1}$	$\frac{1}{n}\sum_{j=1}^n H\left(i,j\right) ^{-1}=\frac{1}{n}\sum_{k=1}^n\sum_{j=1}^na_k\left(i,j\right) {x_k}^{-1}=\frac{1}{n}\sum_{k=1}^nx_k^{-1}\sum_{j=1}^na_k\left(i,j\right) =\frac{1}{i}\sum_{k=1}^ix_k^{-1}$	$\frac{1}{n}\sum_{j=1}^n H(i,j)^{-1} = \frac{1}{n}\sum_{k=1}^n \sum_{j=1}^n a_k(i,j)x_k^{-1} = \frac{1}{n}\sum_{k=1}^n x_k^{-1}\sum_{j=1}^n a_k(i,j) = \frac{1}{i}\sum_{k=1}^i x_k^{-1}$	<div>✖</div>
$a_k(i,j)=\binom{n-i}{j-k}\binom{i-1}{k-1}\Big/\binom{n-1}{j-1}=\frac{(n-i)!(n-j)!(i-1)!(j-1)!}{(n-1)!(k-1)!(n-i-j+k)!(i-k)!(j-k)!}$	$a_k(i,j)=\binom{n-i}{j-k}\binom{i-1}{k-1}\Big/\binom{n-1}{j-1}=\frac{(n-i)!(n-j)!(i-1)!(j-1)!}{(n-1)!(k-1)!(n-i-j+k)!(i-k)!(j-k)!}$	$a_k(i,j)=\binom{n-i}{j-k}\binom{i-1}{k-1}\Big/\binom{n-1}{j-1}=\frac{(n-i)!(n-j)!(i-1)!(j-1)!}{(n-1)!(k-1)!(n-i-j+k)!(i-k)!(j-k)!}$	$a_k\left(i,j\right) =\binom{n-i}{j-k}\binom{i-1}{k-1}\Big/\binom{n-1}{j-1}=\frac{(n-i)!\left(n-j\right) !\left(i-1\right) !\left(j-1\right) !}{\left(n-1\right) !\left(k-1\right) !\left(n-i-j+k\right) !\left(i-k\right) !\left(j-k\right) !}$	$a_k(i,j)=\binom{n-i}{j-k}\binom{i-1}{k-1}\Big/\binom{n-1}{j-1}=\frac{(n-i)!(n-j)!(i-1)!(j-1)!}{(n-1)!(k-1)!(n-i-j+k)!(i-k)!(j-k)!}$	<div>✖</div>
$\left(x_1\cdot\frac{x_1+x_2}{2}\dots\frac{x_1+x_2+\dots x_n}{n}\right)^{1/n}\geq\frac{1}{n}(x_1+\sqrt{x_1x_2}+\dots+\sqrt[n]{x_1x_2\dots x_n})$	$\left(x_1\cdot\frac{x_1+x_2}{2}\dots\frac{x_1+x_2+\dots x_n}{n}\right)^{1/n}\geq\frac{1}{n}(x_1+\sqrt{x_1x_2}+\dots+\sqrt[n]{x_1x_2\dots x_n})$	$\left(x_1\cdot\frac{x_1+x_2}{2}\dots\frac{x_1+x_2+\dots x_n}{n}\right)^{1/n}\geq\frac{1}{n}(x_1+\sqrt{x_1x_2}+\dots+\sqrt[n]{x_1x_2\dots x_n})$	$\left(x_1\cdot\frac{x_1+x_2}{2}\dots\frac{x_1+x_2+\dots x_n}{n}\right)^{1/n}\geq\frac{1}{n}(x_1+\sqrt{x_1x_2}+\dots+\sqrt[n]{x_1x_2\dots x_n})$	<div>$\left(\mathrm{x}_{\mathrm{1}}\cdot\mathrm{frac{\mathrm{x}_{\mathrm{1}}+\mathrm{x}_{\mathrm{2}}}{2}}\cdot\mathrm{frac{\mathrm{x}_{\mathrm{1}}+\mathrm{x}_{\mathrm{2}}+\mathrm{x}_{\mathrm{3}}}{3}}\cdot\mathrm{frac{\mathrm{x}_{\mathrm{1}}+\mathrm{x}_{\mathrm{2}}+\mathrm{x}_{\mathrm{3}}+\mathrm{x}_{\mathrm{4}}}{4}}\cdot\mathrm{frac{\mathrm{x}_{\mathrm{1}}+\mathrm{x}_{\mathrm{2}}+\mathrm{x}_{\mathrm{3}}+\mathrm{x}_{\mathrm{4}}+\mathrm{x}_{\mathrm{5}}}{5}}\cdot\mathrm{frac{\mathrm{x}_{\mathrm{1}}+\mathrm{x}_{\mathrm{2}}+\mathrm{x}_{\mathrm{3}}+\mathrm{x}_{\mathrm{4}}+\mathrm{x}_{\mathrm{5}}+\mathrm{x}_{\mathrm{6}}}{6}}\cdot\mathrm{frac{\mathrm{x}_{\mathrm{1}}+\mathrm{x}_{\mathrm{2}}+\mathrm{x}_{\mathrm{3}}+\mathrm{x}_{\mathrm{4}}+\mathrm{x}_{\mathrm{5}}+\mathrm{x}_{\mathrm{6}}+\mathrm{x}_{\mathrm{7}}}{7}}\cdot\mathrm{frac{\mathrm{x}_{\mathrm{1}}+\mathrm{x}_{\mathrm{2}}+\mathrm{x}_{\mathrm{3}}+\mathrm{x}_{\mathrm{4}}+\mathrm{x}_{\mathrm{5}}+\mathrm{x}_{\mathrm{6}}+\mathrm{x}_{\mathrm{7}}+\mathrm{x}_{\mathrm{8}}}{8}}\cdot\mathrm{frac{\mathrm{x}_{\mathrm{1}}+\mathrm{x}_{\mathrm{2}}+\mathrm{x}_{\mathrm{3}}+\mathrm{x}_{\mathrm{4}}+\mathrm{x}_{\mathrm{5}}+\mathrm{x}_{\mathrm{6}}+\mathrm{x}_{\mathrm{7}}+\mathrm{x}_{\mathrm{8}}+\mathrm{x}_{\mathrm{9}}}{9}}\cdot\mathrm{frac{\mathrm{x}_{\mathrm{1}}+\mathrm{x}_{\mathrm{2}}+\mathrm{x}_{\mathrm{3}}+\mathrm{x}_{\mathrm{4}}+\mathrm{x}_{\mathrm{5}}+\mathrm{x}_{\mathrm{6}}+\mathrm{x}_{\mathrm{7}}+\mathrm{x}_{\mathrm{8}}+\mathrm{x}_{\mathrm{9}}+\mathrm{x}_{\mathrm{10}}}{10}}\right)^{1/10}\geq\frac{1}{10}\left(\mathrm{x}_{\mathrm{1}}+\sqrt{\mathrm{x}_{\mathrm{1}}\mathrm{x}_{\mathrm{2}}}+\sqrt[3]{\mathrm{x}_{\mathrm{1}}\mathrm{x}_{\mathrm{2}}\mathrm{x}_{\mathrm{3}}}+\sqrt[4]{\mathrm{x}_{\mathrm{1}}\mathrm{x}_{\mathrm{2}}\mathrm{x}_{\mathrm{3}}\mathrm{x}_{\mathrm{4}}}+\sqrt[5]{\mathrm{x}_{\mathrm{1}}\mathrm{x}_{\mathrm{2}}\mathrm{x}_{\mathrm{3}}\mathrm{x}_{\mathrm{4}}\mathrm{x}_{\mathrm{5}}}+\sqrt[6]{\mathrm{x}_{\mathrm{1}}\mathrm{x}_{\mathrm{2}}\mathrm{x}_{\mathrm{3}}\mathrm{x}_{\mathrm{4}}\mathrm{x}_{\mathrm{5}}\mathrm{x}_{\mathrm{6}}}+\sqrt[7]{\mathrm{x}_{\mathrm{1}}\mathrm{x}_{\mathrm{2}}\mathrm{x}_{\mathrm{3}}\mathrm{x}_{\mathrm{4}}\mathrm{x}_{\mathrm{5}}\mathrm{x}_{\mathrm{6}}\mathrm{x}_{\mathrm{7}}}+\sqrt[8]{\mathrm{x}_{\mathrm{1}}\mathrm{x}_{\mathrm{2}}\mathrm{x}_{\mathrm{3}}\mathrm{x}_{\mathrm{4}}\mathrm{x}_{\mathrm{5}}\mathrm{x}_{\mathrm{6}}\mathrm{x}_{\mathrm{7}}\mathrm{x}_{\mathrm{8}}}+\sqrt[9]{\mathrm{x}_{\mathrm{1}}\mathrm{x}_{\mathrm{2}}\mathrm{x}_{\mathrm{3}}\mathrm{x}_{\mathrm{4}}\mathrm{x}_{\mathrm{5}}\mathrm{x}_{\mathrm{6}}\mathrm{x}_{\mathrm{7}}\mathrm{x}_{\mathrm{8}}\mathrm{x}_{\mathrm{9}}}\right)$</div>	<div>✖</div>
$v_k=\begin{cases}0^{r_k}, & \text{if } 0\leq r_k\leq P, \\ \omega 0^{r_k-1}, & \text{if } P< r_k< d_{k-1}.\end{cases}$	$v_k=\begin{cases}0^{r_k}, & \text{if } 0\leq r_k\leq P, \\ \omega 0^{r_k-1}, & \text{if } P< r_k< d_{k-1}.\end{cases}$	$v_k=\begin{cases}0^{r_k}, & \text{if } 0\leq r_k\leq P, \\ \omega 0^{r_k-1}, & \text{if } P< r_k< d_{k-1}.\end{cases}$	${\bf v}_{\bf k}=\begin{cases}0^{r_{\bf k}}, & \text{if } 0\leqslant r_{\bf k}\leqslant {\bf P}, \\ \omega 0^{r_{\bf k}-1}, & \text{if } {\bf P}<r_{\bf k}<d_{{\bf k}-1}.\end{cases}$	$v_k=\begin{cases}0^{r_k}, & \text{if } 0\leq r_k\leq P, \\ \omega 0^{r_k-1}, & \text{if } P< r_k< d_{k-1}.\end{cases}$	$v_k=\begin{cases}0^{r_k}, & if 0\leq r_k\leq P, \\ \omega 0^{r_k-1}, & if P< r_k< d_{k-1}.\end{cases}$

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$\hat{\boldsymbol{\beta}} = \underbrace{\left(\mathbf{Z}_X^\top \mathbf{Z}_X + \lambda \mathbf{I}_R\right)}_{\mathbf{A}}^{-1} \mathbf{Z}_X^\top \mathbf{y}.$	$\hat{\boldsymbol{\beta}} = \underbrace{\left(\mathbf{Z}_X^\top \mathbf{Z}_X + \lambda \mathbf{I}_R\right)}_{\mathbf{A}}^{-1} \mathbf{Z}_X^\top \mathbf{y}.$	$\hat{\boldsymbol{\beta}} = \underbrace{\left(\mathbf{Z}_X^\top \mathbf{Z}_X + \lambda \mathbf{I}_R\right)}_{\mathbf{A}}^{-1} \mathbf{Z}_X^\top \mathbf{y}.$	<div>$\hat{\boldsymbol{\beta}} = \left(\underbrace{\mathbf{Z}_X^\top \mathbf{Z}_X + \lambda \mathbf{I}_R}_{\mathbf{A}}\right)^{-1} \mathbf{Z}_X^\top \mathbf{y}.$</div>	$\hat{\boldsymbol{\beta}} = \underbrace{\left(\mathbf{Z}_X^\top \mathbf{Z}_X + \lambda \mathbf{I}_R\right)}_{\mathbf{A}}^{-1} \mathbf{Z}_X^\top \mathbf{y}.$	✖
$\mathbb{E}[\mathbf{f}_*] = \mathbf{K}_{X_*,X} \overbrace{[\sigma^2 \mathbf{I} + \mathbf{K}_{X,X}]^{-1} \mathbf{y}}^{\beta},$	$\mathbb{E}[\mathbf{f}_*] = \mathbf{K}_{X_*,X} \overbrace{[\sigma^2 \mathbf{I} + \mathbf{K}_{X,X}]^{-1} \mathbf{y}}^{\beta},$	$\mathbb{E}[\mathbf{f}_*] = \mathbf{K}_{X_*,X} \overbrace{[\sigma^2 \mathbf{I} + \mathbf{K}_{X,X}]^{-1} \mathbf{y}}^{\beta},$	<div>$\mathbb{E}[\mathbf{f}_*] = \mathbf{K}_{X_*,X} \overbrace{[\sigma^2 \mathbf{I} + \mathbf{K}_{X,X}]^{-1} \mathbf{y}}^{\beta}$</div>	$\mathbb{E}[\mathbf{f}_*] = \mathbf{K}_{X_*,X} \overbrace{\mathbf{I} + \mathbf{K}_{X,X} \mathbf{y} \mathbf{I}}^{\beta},$	✖
$\omega_{\bar{z}^i \bar{t}} = \frac{1}{ \bar{I}_r \bar{\gamma} } \sum_{\omega_{\bar{z}^i} \in \bar{I}_r} \omega_{\bar{z}^i} - \frac{1}{ \bar{I}_{r_i} \bar{\gamma} } \sum_{\omega_{\bar{z}^i} \in \bar{I}_{r_i}} \omega_{\bar{k}^i}.$	$\omega_{\bar{z}^i \bar{t}} = \frac{1}{ \bar{I}_r \bar{\gamma} } \sum_{\omega_{\bar{z}^i} \in \bar{I}_r} \omega_{\bar{z}^i} - \frac{1}{ \bar{I}_{r_i} \bar{\gamma} } \sum_{\omega_{\bar{z}^i} \in \bar{I}_{r_i}} \omega_{\bar{k}^i}.$	$\omega_{\bar{z}^i \bar{t}} = \frac{1}{ \bar{I}_r \bar{\gamma} } \sum_{\omega_{\bar{z}^i} \in \bar{I}_r} \omega_{\bar{z}^i} - \frac{1}{ \bar{I}_{r_i} \bar{\gamma} } \sum_{\omega_{\bar{z}^i} \in \bar{I}_{r_i}} \omega_{\bar{k}^i}.$	<div>$\omega_{\bar{z}^i \bar{t}} = \frac{1}{ \bar{I}_r \bar{\gamma} } \sum_{\omega_{\bar{z}^i} \in \bar{I}_r} \omega_{\bar{z}^i} - \frac{1}{ \bar{I}_{r_i} \bar{\gamma} } \sum_{\omega_{\bar{z}^i} \in \bar{I}_{r_i}} \omega_{\bar{k}^i}.$</div>	$\omega_{\bar{z}^i \bar{t}} = \frac{1}{ \bar{I}_r \bar{\gamma} } \sum_{\omega_{\bar{z}^i} \in \bar{I}_r} \omega_{\bar{z}^i} - \frac{1}{ \bar{I}_{r_i} \bar{\gamma} } \sum_{\omega_{\bar{z}^i} \in \bar{I}_{r_i}} \omega_{\bar{k}^i}.$	$\omega_{z_i \bar{t}} = \frac{1}{ I_r \gamma } \sum_{\omega_i \in I_r} \omega_{z_i} - \frac{1}{ I_{r_i} \gamma } \sum_{\omega_i \in I_{r_i}} \omega_{\bar{k}^i}$
$\mathbf{z}(\mathbf{x}) = \begin{bmatrix} \frac{1}{\sqrt{R}} z_{\omega_1}(\mathbf{x}) \\ \frac{1}{\sqrt{R}} z_{\omega_2}(\mathbf{x}) \\ \vdots \\ \frac{1}{\sqrt{R}} z_{\omega_R}(\mathbf{x}) \end{bmatrix}.$	$\mathbf{z}(\mathbf{x}) = \begin{bmatrix} \frac{1}{\sqrt{R}} z_{\omega_1}(\mathbf{x}) \\ \frac{1}{\sqrt{R}} z_{\omega_2}(\mathbf{x}) \\ \vdots \\ \frac{1}{\sqrt{R}} z_{\omega_R}(\mathbf{x}) \end{bmatrix}.$	$\mathbf{z}(\mathbf{x}) = \begin{bmatrix} \frac{1}{\sqrt{R}} z_{\omega_1}(\mathbf{x}) \\ \frac{1}{\sqrt{R}} z_{\omega_2}(\mathbf{x}) \\ \vdots \\ \frac{1}{\sqrt{R}} z_{\omega_R}(\mathbf{x}) \end{bmatrix}.$	<div>$\mathbf{x}(\mathbf{x}) = \left(\frac{1}{\sqrt{n}} z_{\omega_1}(\mathbf{x}) \frac{1}{\sqrt{R}} z_{\omega_1}(\mathbf{x}) \vdots \sqrt{z_{\omega_k}(\mathbf{x})}\right).$</div>	<div>$\mathbf{z}(\mathbf{x}) = \begin{bmatrix} \frac{1}{\sqrt{R}} z_{\omega_1}(\mathbf{x}) \\ \frac{1}{\sqrt{R}} z_{\omega_2}(\mathbf{x}) \\ \vdots \\ \frac{1}{\sqrt{R}} z_{\omega_R}(\mathbf{x}) \end{bmatrix}.$</div>	✖
$\mathbf{K}_X \approx \begin{bmatrix} \mathbf{z}(\mathbf{x}_1) \\ \vdots \\ \mathbf{z}(\mathbf{x}_N) \end{bmatrix} \begin{bmatrix} \mathbf{z}(\mathbf{x}_1) & \dots & \mathbf{z}(\mathbf{x}_N) \end{bmatrix} = \mathbf{Z}_X \mathbf{Z}_X^\top$	$\mathbf{K}_X \approx \begin{bmatrix} \mathbf{z}(\mathbf{x}_1) \\ \vdots \\ \mathbf{z}(\mathbf{x}_N) \end{bmatrix} \begin{bmatrix} \mathbf{z}(\mathbf{x}_1) & \dots & \mathbf{z}(\mathbf{x}_N) \end{bmatrix} = \mathbf{Z}_X \mathbf{Z}_X^\top$	$\mathbf{K}_X \approx \begin{bmatrix} \mathbf{z}(\mathbf{x}_1) \\ \vdots \\ \mathbf{z}(\mathbf{x}_N) \end{bmatrix} \begin{bmatrix} \mathbf{z}(\mathbf{x}_1) & \dots & \mathbf{z}(\mathbf{x}_N) \end{bmatrix} = \mathbf{Z}_X \mathbf{Z}_X^\top$	✖	$\mathbf{K}_X \approx \begin{bmatrix} \mathbf{z}(\mathbf{x}_1) \\ \vdots \\ \mathbf{z}(\mathbf{x}_N) \end{bmatrix} \begin{bmatrix} \mathbf{z}(\mathbf{x}_1) \cdots \mathbf{z}(\mathbf{x}_N) \end{bmatrix} = \mathbf{Z}_X \mathbf{Z}_X^\top$	$\mathbf{K}_\mathbf{X} \approx \begin{bmatrix} z\left(x_1\right) \\ 2\left(\mathbf{x}_\mathbf{N}\right) \end{bmatrix} \left[\left(\mathbf{x}_1\right) \cdots \quad z\left(\mathbf{x}_\mathbf{N}\right)\right]=\mathbf{Z}_\mathbf{x} \mathbf{Z}_\mathbf{x}^{\mathrm{T}}$
$\widehat{\mu}_n \; = \; \frac{1}{n} \sum_{i=1}^n X_i, \quad and \quad \widehat{\mu}_n - \mathbb{E} \widehat{\mu}_n := \frac{1}{n} \sum_{i=1}^n \mathbb{Z}_i = \frac{1}{n} \sum_{i=1}^n X_i - \mathbb{E}(X_i).$	$\widehat{\mu}_n \; = \; \frac{1}{n} \sum_{i=1}^n X_i, \quad and \quad \widehat{\mu}_n - \mathbb{E} \widehat{\mu}_n := \frac{1}{n} \sum_{i=1}^n \mathbb{Z}_i = \frac{1}{n} \sum_{i=1}^n X_i - \mathbb{E}(X_i).$	$\widehat{\mu}_n \; = \; \frac{1}{n} \sum_{i=1}^n X_i, \quad and \quad \widehat{\mu}_n - \mathbb{E} \widehat{\mu}_n := \frac{1}{n} \sum_{i=1}^n \mathbb{Z}_i = \frac{1}{n} \sum_{i=1}^n X_i - \mathbb{E}(X_i).$	✖	$\widehat{\mu}_n \; = \; \frac{1}{n} \sum_{i=1}^n X_i, \quad and \quad \widehat{\mu}_n - \mathbb{E} \widehat{\mu}_n := \frac{1}{n} \sum_{i=1}^n \mathbb{Z}_i = \frac{1}{n} \sum_{i=1}^n X_i - \mathbb{E}(X_i).$	$and \quad \widehat{\mu}_n - E \hat{a}_n : \frac{1}{n} \sum_{i=1}^n \mathbb{Z}_i = \frac{1}{n} \sum_{i=1}^n X_i - E\left(X_i\right)$
$\gamma := \Delta^2/p, \text{ where } \Delta^2 := \sum_{k=1}^p (\mu_k^{(1)} - \mu_k^{(2)})^2 \text{ and } \mu^{(i)} = (\mu_1^{(i)}, \dots, \mu_p^{(i)}) \in \mathbb{R}^p, i = 1, 2.$	$\gamma := \Delta^2/p, \text{ where } \Delta^2 := \sum_{k=1}^p (\mu_k^{(1)} - \mu_k^{(2)})^2 \text{ and } \mu^{(i)} = (\mu_1^{(i)}, \dots, \mu_p^{(i)}) \in \mathbb{R}^p, i = 1, 2.$	$\gamma := \Delta^2/p, \text{ where } \Delta^2 := \sum_{k=1}^p (\mu_k^{(1)} - \mu_k^{(2)})^2 \text{ and } \mu^{(i)} = (\mu_1^{(i)}, \dots, \mu_p^{(i)}) \in \mathbb{R}^p, i = 1, 2.$	<div>$\gamma := \Delta^2/p, \text{ where } \Delta^2 := \sum_{k=1}^p \left(\mu_k^{(1)} - \mu_k^{(2)}\right)^2 \text{ and } \mu^{(i)} = \left(\mu_1^{(i)}, \dots, \mu_p^{(i)}\right) \in \mathbb{R}^p, i = 1, 2.$</div>	<div>$\gamma := \Delta^2/p, \text{ where } \Delta^2 := \sum_{k=1}^p (\mu_k^{(1)} - \mu_k^{(2)})^2 \text{ and } \mu^{(i)} = (\mu_1^{(i)}, \dots, \mu_p^{(i)}) \in \mathbb{R}^p, i = 1, 2.$</div>	✖

Image	ours	SimpleTex	阿里达摩院	pix2text	新东方																																																																	
<div>$\begin{aligned} \left \left\langle P_2(\widehat{\Sigma}_Y - \Sigma_Y)P_2, \widehat{Z} - Z^* \right\rangle \right &\leq \left \left\langle P_2\Psi P_2, \widehat{Z} - Z^* \right\rangle \right + \left \left\langle P_2P_1\Psi P_1P_2, \widehat{Z} - Z^* \right\rangle \right + \\ &\quad 2 \left \left\langle P_2P_1\Psi P_2, \widehat{Z} - Z^* \right\rangle \right \\ &\leq \frac{1}{n} \left\ \widehat{Z} - Z^* \right\ _1 \ \Psi\ _2 \left(1 + (w_1 - w_2)^2 + 2 w_2 - w_1 \right) \\ &\leq \frac{1}{n} \left\ \widehat{Z} - Z^* \right\ _1 \ \Psi\ _2 \left(1 + w_1 - w_2 \right)^2. \end{aligned}$</div>	<div>$\begin{aligned} \left \left\langle P_2(\widehat{\Sigma}_Y - \Sigma_Y)P_2, \widehat{Z} - Z^* \right\rangle \right &\leq \frac{\left \left\langle P_2\Psi P_2, \widehat{Z} - Z^* \right\rangle \right + \left \left\langle P_2P_1\Psi P_1P_2, \widehat{Z} - Z^* \right\rangle \right + 2 \left \left\langle P_2P_1\Psi P_2, \widehat{Z} - Z^* \right\rangle \right }{2} \\ &\leq \frac{1}{n} \left\ \widehat{Z} - Z^* \right\ _1 \ \Psi\ _2 \left(1 + (w_1 - w_2)^2 + 2 w_2 - w_1 \right) \\ &\leq \frac{1}{n} \left\ \widehat{Z} - Z^* \right\ _1 \ \Psi\ _2 \left(1 + w_1 - w_2 \right)^2. \end{aligned}$</div>	<div>$\begin{aligned} \left \texttt{\textbackslash angle} P_2(\widehat{\Sigma}_Y - \Sigma_Y)P_2, \widehat{Z} - Z^* \right\rangle \right &\leq \left \left\langle P_2\Psi P_2, \widehat{Z} - Z^* \right\rangle \right + \left \left\langle P_2P_1\Psi P_1P_2, \widehat{Z} - Z^* \right\rangle \right + \\ &\quad 2 \left \left\langle P_2P_1\Psi P_2, \widehat{Z} - Z^* \right\rangle \right \\ &\leq \frac{1}{n} \left\ \widehat{Z} - Z^* \right\ _1 \ \Psi\ _2 \left(1 + (w_1 - w_2)^2 + 2 w_2 - w_1 \right) \\ &\leq \frac{1}{n} \left\ \widehat{Z} - Z^* \right\ _1 \ \Psi\ _2 \left(1 + w_1 - w_2 \right)^2. \end{aligned}$</div> <div>● 本系列模型训练数据均来自公开数据集，训练过程中未使用任何人工标注数据，模型训练完全基于无监督学习，所有标注均由模型自动标注生成。</div>	<div>$\begin{aligned} \left\langle P_2(\widehat{\Sigma}_Y - \Sigma_Y)P_2, \widehat{Z} - Z^* \right\rangle \right &\leq \left \left\langle P_2\Psi P_2, \widehat{Z} - Z^* \right\rangle \right + \left \left\langle P_2P_1\Psi P_1P_2, \widehat{Z} - Z^* \right\rangle \right + \\ &\quad 2 \left \left\langle P_2P_1\Psi P_2, \widehat{Z} - Z^* \right\rangle \right \\ &\leq \frac{1}{n} \left\ \widehat{Z} - Z^* \right\ _1 \ \Psi\ _2 \left(1 + (w_1 - w_2)^2 + 2 w_2 - w_1 \right) \\ &\leq \frac{1}{n} \left\ \widehat{Z} - Z^* \right\ _1 \ \Psi\ _2 \left(1 + w_1 - w_2 \right)^2. \end{aligned}$</div>	<div>$\begin{aligned} \mathbb{P} \left(\sum_{j=1}^q L_j^* > \tau \right) &\leq \mathbb{P} \left(\exists J \in [n], J = q, \sum_{j \in J} L_j > \tau \right) \\ &= \mathbb{P} \left(\max_{J \in [n], J =q} \max_{u=(u_1, \dots, u_q) \in \{-1, 1\}^q} \sum_{j \in J} u_j L_j > \tau \right) \\ &\leq \sum_{J: J =q} \sum_{u \in \{-1, 1\}^q} \mathbb{P} \left(\sum_{j \in J} u_j L_j / \Delta > \tau / \Delta \right) \\ &\leq \binom{n}{q} 2^q \exp \left(- \frac{(\tau / \Delta)^2}{C_1 C_0^2 (\max_j \ R_j \mu\ _2^2) q} \right), \end{aligned}$</div>	<div>$\begin{aligned} \mathbb{P} \left(\sum_{j=1}^q L_j^* > \tau \right) &\leq \mathbb{P} \left(\exists J \in [n], J = q, \sum_{j \in J} L_j > \tau \right) \\ &= \mathbb{P} \left(\max_{J \in [n], J =q} \max_{u=(u_1, \dots, u_q) \in \{-1, 1\}^q} \sum_{j \in J} u_j L_j > \tau \right) \\ &\leq \sum_{J \in \{1, j\}=q, n \in \{-1, \dots, u_j\} \times \mathbb{P} \left(\sum_{j \in J} u_j L_j / \Delta > \tau / \Delta \right) \\ &\leq \binom{n}{q} 2^q \exp \left(- C_1 C_0^2 (\max_j \ L_j \mu\ _2^q) q \right), \end{aligned}$</div>	<div>$\begin{aligned} \left\langle P_2\Psi, \widehat{Z} - Z^* \right\rangle &= \text{tr}((\widehat{Z} - Z^*)P_2\Psi) = \text{tr}(P_2\Psi(\widehat{Z} - Z^*)) = \left\langle \Psi, (\widehat{Z} - Z^*)P_2 \right\rangle \\ &= \frac{1}{n} (u_2^T \Psi (\widehat{Z} - Z^*) u_2) = \left\langle \Psi u_2, (\widehat{Z} - Z^*) u_2 / n \right\rangle \\ &= 2 \sum_{k=1}^n \left(\sum_{t \in \mathcal{C}_1} \Psi_{kt} - \sum_{t \in \mathcal{C}_2} \Psi_{kt} \right) \cdot \frac{1}{2n} \left(\sum_{j \in \mathcal{C}_1} (\widehat{Z} - Z^*)_{kj} - \sum_{j \in \mathcal{C}_2} (\widehat{Z} - Z^*)_{kj} \right) \\ &= 2 \left(\sum_{k \in \mathcal{C}_1} \Psi_{kk} (-\widehat{w}_k) - \sum_{t \in \mathcal{C}_2} \Psi_{tt} \widehat{w}_t \right) + 2 \left(\sum_{j \in \mathcal{C}_1} Q_j (-\widehat{w}_j) + \sum_{j \in \mathcal{C}_2} Q_j \widehat{w}_j \right) = Q_{\text{diag}} + Q_{\text{offd}}. \end{aligned}$</div>	<div>$\begin{aligned} \left\langle P_2\Psi, \widehat{Z} - Z^* \right\rangle &= \text{tr}((\widehat{Z} - Z^*)P_2\Psi) = \text{tr}(P_2\Psi(\widehat{Z} - Z^*)) = \left\langle \Psi, (\widehat{Z} - Z^*)P_2 \right\rangle \\ &= \frac{1}{n} (u_2^T \Psi (\widehat{Z} - Z^*) u_2) = \left\langle \Psi u_2, (\widehat{Z} - Z^*) u_2 / n \right\rangle \\ &= 2 \sum_{k=1}^n \left(\sum_{t \in \mathcal{C}_1} \Psi_{kt} - \sum_{t \in \mathcal{C}_2} \Psi_{kt} \right) \cdot \frac{1}{2n} \left(\sum_{j \in \mathcal{C}_1} (\widehat{Z} - Z^*)_{kj} - \sum_{j \in \mathcal{C}_2} (\widehat{Z} - Z^*)_{kj} \right) \\ &= 2 \left(\sum_{k \in \mathcal{C}_1} \Psi_{kk} (-\widehat{w}_k) - \sum_{t \in \mathcal{C}_2} \Psi_{tt} \widehat{w}_t \right) + 2 \left(\sum_{j \in \mathcal{C}_1} Q_j (-\widehat{w}_j) + \sum_{j \in \mathcal{C}_2} Q_j \widehat{w}_j \right) = Q_{\text{diag}} + Q_{\text{offd}}. \end{aligned}$</div>	<div>$\begin{aligned} \left\langle P_2\Psi, \widehat{Z} - Z^* \right\rangle &= \text{tr}((\widehat{Z} - Z^*)P_2\Psi) = \text{tr}(P_2\Psi(\widehat{Z} - Z^*)) = \left\langle \Psi, (\widehat{Z} - Z^*)P_2 \right\rangle \\ &= \frac{1}{n} (u_2^T \Psi (\widehat{Z} - Z^*) u_2) = \left\langle \Psi u_2, (\widehat{Z} - Z^*) u_2 / n \right\rangle \\ &= 2 \sum_{k=1}^n \left(\sum_{t \in \mathcal{C}_1} \Psi_{kt} - \sum_{t \in \mathcal{C}_2} \Psi_{kt} \right) \cdot \frac{1}{2n} \left(\sum_{j \in \mathcal{C}_1} (\widehat{Z} - Z^*)_{kj} - \sum_{j \in \mathcal{C}_2} (\widehat{Z} - Z^*)_{kj} \right) \\ &= 2 \left(\sum_{k \in \mathcal{C}_1} \Psi_{kk} (-\widehat{w}_k) - \sum_{t \in \mathcal{C}_2} \Psi_{tt} \widehat{w}_t \right) + 2 \left(\sum_{j \in \mathcal{C}_1} Q_j (-\widehat{w}_j) + \sum_{j \in \mathcal{C}_2} Q_j \widehat{w}_j \right) = Q_{\text{diag}} + Q_{\text{offd}}. \end{aligned}$</div>																																																														
<div>$\begin{aligned} \mathbb{P} \left(\left \sum_{i=1}^n \sum_{j \neq i}^n \left\langle \mathbb{Z}_i, \mathbb{Z}_j \right\rangle a_{ij} \right > \tau_q \right) &\leq 2 \exp \left(-c \min \left(\frac{\tau_q^2}{(C_0 \max_i \ H_i\ _2)^4 p \ A\ _F^2}, \frac{\tau_q}{(C_0 \max_i \ H_i\ _2)^2 \ A\ _2} \right) \right) \\ &\leq 2 \exp \left(-c \min \left(\frac{(C_4 q \sqrt{np} \log(2en/q))^2}{pnq}, \frac{C_4 q \sqrt{nq} \log(2en/q)}{\sqrt{qn}} \right) \right) \\ &\leq 2 \exp \left(-c (C_4^2 \wedge C_4) q \log(2en/q) \right), \end{aligned}$</div>	<div>$\begin{aligned} \mathbb{P} \left(\left \sum_{i=1}^n \sum_{j \neq i}^n \left\langle \mathbb{Z}_i, \mathbb{Z}_j \right\rangle a_{ij} \right > \tau_q \right) &\leq 2 \exp \left(-c \min \left(\frac{\tau_q^2}{(C_0 \max_i \ H_i\ _2)^4 p \ A\ _F^2}, \frac{\tau_q}{(C_0 \max_i \ H_i\ _2)^2 \ A\ _2} \right) \right) \\ &\leq 2 \exp \left(-c \min \left(\frac{(C_4 q \sqrt{np} \log(2en/q))^2}{pnq}, \frac{C_4 q \sqrt{nq} \log(2en/q)}{\sqrt{qn}} \right) \right) \\ &\leq 2 \exp \left(-c (C_4^2 \wedge C_4) q \log(2en/q) \right), \end{aligned}$</div>	<div>$\begin{aligned} \mathbb{P} \left(\left \sum_{i=1}^n \sum_{j \neq i}^n \left\langle \mathbb{Z}_i, \mathbb{Z}_j \right\rangle a_{ij} \right > \tau_q \right) &\leq 2 \exp \left(-c \min \left(\frac{\tau_q^2}{(C_0 \max_i \ H_i\ _2)^4 p \ A\ _F^2}, \frac{\tau_q}{(C_0 \max_i \ H_i\ _2)^2 \ A\ _2} \right) \right) \\ &\leq 2 \exp \left(-c \min \left(\frac{(C_4 q \sqrt{np} \log(2en/q))^2}{pnq}, \frac{C_4 q \sqrt{nq} \log(2en/q)}{\sqrt{qn}} \right) \right) \\ &\leq 2 \exp \left(-c (C_4^2 \wedge C_4) q \log(2en/q) \right), \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \texttt{\textbackslash begin{array} \textbackslash tr l l \{\{\}\} \& \{\{\textbackslash P_{-12}\} \Psi \textbackslash \widehat{\textbackslash char{Z}}{-Z^*}\} \} \textbackslash \mathrm{tr} \{\textbackslash C_{\textbackslash diag} \textbackslash offd\} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{array} \textbackslash tr l l \{\{\}\} \& \{\{\textbackslash P_{-12}\} \Psi \textbackslash \widehat{\textbackslash char{Z}}{-Z^*}\} \} \textbackslash \mathrm{tr} \{\textbackslash C_{\textbackslash diag} \textbackslash offd\} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] 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\{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} 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[0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] \end{aligned}$</div>	<div>$\begin{aligned} \left[\begin{array}{l} \textbackslash begin{aligned} \{\{\}\} \& \{\{\} \{\{\textbackslash bf E\} \textbackslash \biguplus \textbackslash operatorname{ } \{s u p\}_{s \in [0, \textbackslash tau_{\textbackslash n} \textbackslash t} \end{array} \right] 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Image

$$\mathbb{U}_n=\left\{\begin{pmatrix}1&*&\cdots&*&*\\0&1&\cdots&*&*\\ \vdots&\vdots&&\vdots&\vdots\\0&0&\cdots&1&*\\0&0&\cdots&0&1\end{pmatrix}\right\}.$$

$$\begin{aligned}k(\mathbf{x},\mathbf{y})&=k(\mathbf{x}-\mathbf{y})\\&=\int p(\omega)\exp(i\omega^\top(\mathbf{x}-\mathbf{y}))\mathrm{d}\omega\\&=\mathbb{E}_\omega\big[\exp(i\omega^\top(\mathbf{x}-\mathbf{y}))\big]\\&\approx\frac{1}{R}\sum_{r=1}^R\exp(i\omega_r^\top(\mathbf{x}-\mathbf{y}))\\&=\begin{bmatrix}\frac{1}{\sqrt{R}}\exp(i\omega_1^\top\mathbf{x})\\\frac{1}{\sqrt{R}}\exp(i\omega_2^\top\mathbf{x})\\\vdots\\\frac{1}{\sqrt{R}}\exp(i\omega_R^\top\mathbf{x})\end{bmatrix}^\top\begin{bmatrix}\frac{1}{\sqrt{R}}\exp(-i\omega_1^\top\mathbf{y})\\\frac{1}{\sqrt{R}}\exp(-i\omega_2^\top\mathbf{y})\\\vdots\\\frac{1}{\sqrt{R}}\exp(-i\omega_R^\top\mathbf{y})\end{bmatrix}\\&\triangleq\mathbf{h}(\mathbf{x})\mathbf{h}(\mathbf{y})^*.\end{aligned}$$

$$\begin{aligned}H_{k+1}&=H_k+\frac{(s_k-H_ky_k)v_k^T+v_k(s_k-H_ky_k)^T}{(v_k^Ty_k)^2}-\frac{(s_k-H_ky_k)^Ty_k}{(v_k^Ty_k)^2}v_kv_k^T\\&=H_k+\frac{(z_k-U_kM_k^{-1}U_k^Ty_k)v_k^T+v_k(z_k-U_kM_k^{-1}U_k^Ty_k)^T}{(v_k^Ty_k)^2}-\frac{(s_k-H_ky_k)^Ty_k}{(v_k^Ty_k)^2}v_kv_k^T\\&=H_k+\frac{1}{\rho_k}\begin{bmatrix}U_k&v_k&z_k\\-y_k^TU_kM_k^{-1}&0&1\\0_{1\times2k}&1&0\end{bmatrix}\begin{bmatrix}U_k^T\\v_k^T\\z_k^T\end{bmatrix}-\frac{\theta_k}{\rho_k^2}v_kv_k^T\\(3.6)\quad&=H_0+[U_k\quad v_k\quad z_k]\begin{bmatrix}M_k^{-1}&-\frac{1}{\rho_k}M_k^{-1}U_k^Ty_k&0_{2k\times1}\\\frac{1}{\rho_k}y_k^TU_kM_k^{-1}&-\frac{\theta_k}{\rho_k}&1\\0_{1\times2k}&\frac{1}{\rho_k}&0\end{bmatrix}\begin{bmatrix}U_k^T\\v_k^T\\z_k^T\end{bmatrix}\end{aligned}$$

$$\begin{bmatrix}-(R_k^{YY})^{-T}(R_k+R_k^T-(D_k+Y_k^TH_0Y_k))(R_k^{YY})^{-1}& (R_k^{YY})^{-T}-(R_k^{YY})^{-T}\\(R_k^{VY})^{-1}& 0\\-(R_k^{VY})^{-1}& 0\end{bmatrix}\equiv\begin{bmatrix}(N_k)_{11}(N_k)_{21}-(N_k)_{21}&\\(N_k)_{21}^T& 0& 0\\-(N_k)_{21}^T& 0& 0\end{bmatrix}$$

$$\mathbf{M}=\begin{pmatrix}300&3&3&3\\210&4&2&3\\201&2&2&5\\120&5&1&3\\111&3&1&5\\102&1&1&7\\030&6&0&3\\021&4&0&5\\012&2&0&7\\003&0&0&9\end{pmatrix}.$$

$$\begin{bmatrix}V_k&v_k\end{bmatrix}=V_{k+1}\\\begin{bmatrix}Z_k&z_k\end{bmatrix}=Z_{k+1}=\begin{bmatrix}S_k-H_0Y_k&s_k-H_0y_k\end{bmatrix}=S_{k+1}-H_0Y_{k+1}\\\begin{bmatrix}(M_k)_{11}&0_{k\times1}\\0_{1\times k}&0\end{bmatrix}=\begin{bmatrix}0_{k\times k}&0_{k\times1}\\0_{1\times k}&0\end{bmatrix}=0_{(k+1)\times(k+1)}\\\begin{bmatrix}(M_k)_{21}&0_{k\times1}\\\frac{y_k^TV_k}{\rho_k}\end{bmatrix}=\begin{bmatrix}(R_k^{YY})^T&\\y_k^TV_k&\frac{y_k^Tv_k}{\rho_k}\end{bmatrix}=(R_{k+1}^{YY})^T\\\begin{bmatrix}(M_k)_{12}&V_k^Ty_k\\\frac{0_{1\times k}}{\rho_k}\end{bmatrix}=\begin{bmatrix}(R_k^{YY})&V_k^Ty_k\\y_k^Tv_k&\rho_k\end{bmatrix}=R_{k+1}^{YY}\\\begin{bmatrix}(M_k)_{22}&Z_k^Ty_k\\\frac{y_k^TZ_k}{\beta_k}\end{bmatrix}=\begin{bmatrix}R_k+R_k^T-(D_k+Y_k^TH_0Y_k)&(S_k-H_0Y_k)^Ty_k\\y_k^T(S_k-H_0Y_k)&y_k^T(s_k-H_0y_k)\end{bmatrix}=R_{k+1}+R_{k+1}^T-(D_{k+1}+Y_{k+1}^TH_0Y_{k+1})$$

$$\text{prodUpdate}\left(S_k^TY_k,S_k,Y_k,s_k,y_k\right)\equiv\begin{cases}\begin{bmatrix}S_k^TY_k&S_k^Ty_k\\s_k^TY_k&s_k^Ty_k\end{bmatrix}&\text{if }k<l\\\begin{bmatrix}\overline{\left(S_k^TY_k\right)}&\overline{S_k^Ty_k}\\s_k^TY_k&s_k^Ty_k\end{bmatrix}&\text{if }k\geq l.\end{cases}$$

$$\left\|\left[\tilde{U}_t,\tilde{V}_t\right]\right\|\leq\left\|\begin{pmatrix}t[Y,\tilde{V}]&0\\0&-t[Y^*,\tilde{V}]\end{pmatrix}\right\|+\left\|\begin{pmatrix}0&[(1-t^2YY^*)^{1/2},\tilde{V}]\\[(1-t^2YY^*)^{1/2},\tilde{V}]&0\end{pmatrix}\right\|\leq t'\epsilon+(1-t')\epsilon=\epsilon.$$

Hence \tilde{U}_t,\tilde{V}_t define a $*$ -homomorphism \tilde{f}_t which is a lift of f_t .

$$\left\|\left[\tilde{U}_t,\tilde{V}_t\right]\right\|\leq\left\|\begin{pmatrix}t[Y,\tilde{V}]&0\\0&-t[Y^*,\tilde{V}]\end{pmatrix}\right\|+\left\|\begin{pmatrix}0&[(1-t^2YY^*)^{1/2},\tilde{V}]\\[(1-t^2YY^*)^{1/2},\tilde{V}]&0\end{pmatrix}\right\|\leq t'\epsilon+(1-t')\epsilon=\epsilon.$$

Hence \tilde{U}_t,\tilde{V}_t define a $*$ -homomorphism \tilde{f}_t which is a lift of f_t .

$$\left\|\left[\tilde{U}_t,\tilde{V}_t\right]\right\|\leq\left\|\begin{pmatrix}t[Y,\tilde{V}]&0\\0&-t[Y^*,\tilde{V}]\end{pmatrix}\right\|+\left\|\begin{pmatrix}0&[(1-t^2YY^*)^{1/2},\tilde{V}]\\[(1-t^2YY^*)^{1/2},\tilde{V}]&0\end{pmatrix}\right\|\leq t'\epsilon+(1-t')\epsilon=\epsilon.$$

Hence \tilde{U}_t,\tilde{V}_t define a $*$ -homomorphism \tilde{f}_t which is a lift of f_t .

ours

$$\mathbb{U}_n=\left\{\begin{pmatrix}1&*&\cdots&*&*\\0&1&\cdots&*&*\\ \vdots&\vdots&&\vdots&\vdots\\0&0&\cdots&1&*\\0&0&\cdots&0&1\end{pmatrix}\right\}.$$

$$\begin{aligned}k(\mathbf{x},\mathbf{y})&=k(\mathbf{x}-\mathbf{y})\\&=\int p(\omega)\exp(i\omega^\top(\mathbf{x}-\mathbf{y}))\mathrm{d}\omega\\&=\mathbb{E}_\omega\big[\exp(i\omega^\top(\mathbf{x}-\mathbf{y}))\big]\\&\triangleq\frac{1}{R}\sum_{r=1}^R\exp(i\omega_r^\top(\mathbf{x}-\mathbf{y}))\\&=\begin{bmatrix}\frac{1}{\sqrt{R}}\exp(i\omega_1^\top\mathbf{x})\\\frac{1}{\sqrt{R}}\exp(i\omega_2^\top\mathbf{x})\\\vdots\\\frac{1}{\sqrt{R}}\exp(i\omega_R^\top\mathbf{x})\end{bmatrix}^\top\begin{bmatrix}\frac{1}{\sqrt{R}}\exp(-i\omega_1^\top\mathbf{y})\\\frac{1}{\sqrt{R}}\exp(-i\omega_2^\top\mathbf{y})\\\vdots\\\frac{1}{\sqrt{R}}\exp(-i\omega_R^\top\mathbf{y})\end{bmatrix}\\&\triangleq\mathbf{h}(\mathbf{x})\mathbf{h}(\mathbf{y})^*.\end{aligned}$$

$$\begin{aligned}H_{k+1}&=H_k+\frac{(s_k-H_ky_k)v_k^T+v_k(s_k-H_ky_k)^T}{(v_k^Ty_k)^2}-\frac{(s_k-H_ky_k)^Ty_k}{(v_k^Ty_k)^2}v_kv_k^T\\&=H_k+\frac{(z_k-U_kM_k^{-1}U_k^Ty_k)v_k^T+v_k(z_k-U_kM_k^{-1}U_k^Ty_k)^T}{(v_k^Ty_k)^2}-\frac{(s_k-H_ky_k)^Ty_k}{(v_k^Ty_k)^2}v_kv_k^T\\&=H_k+\frac{1}{\rho_k}\begin{bmatrix}U_k&v_k&z_k\\-y_k^TU_kM_k^{-1}&0&1\\0_{1\times2k}&1&0\end{bmatrix}\begin{bmatrix}U_k^T\\v_k^T\\z_k^T\end{bmatrix}-\frac{\theta_k}{\rho_k^2}v_kv_k^T\\(3.6)\quad&=H_0+[U_k\quad v_k\quad z_k]\begin{bmatrix}M_k^{-1}&-\frac{1}{\rho_k}M_k^{-1}U_k^Ty_k&0_{2k\times1}\\\frac{1}{\rho_k}y_k^TU_kM_k^{-1}&-\frac{\theta_k}{\rho_k}&1\\0_{1\times2k}&\frac{1}{\rho_k}&0\end{bmatrix}\begin{bmatrix}U_k^T\\v_k^T\\z_k^T\end{bmatrix}\end{aligned}$$

$$\begin{bmatrix}(R_k^{YY})^{-T}(R_k+R_k^T-(D_k+Y_k^TH_0Y_k))(R_k^{YY})^{-1}& (R_k^{YY})^{-T}-(R_k^{YY})^{-T}\\(R_k^{VY})^{-1}& 0\\-(R_k^{VY})^{-1}& 0\end{bmatrix}\equiv\begin{bmatrix}(N_k)_{11}(N_k)_{21}-(N_k)_{21}&\\(N_k)_{21}^T& 0& 0\\-(N_k)_{21}^T& 0& 0\end{bmatrix}$$

$$\mathbf{M}=\begin{pmatrix}300&3&3&3\\210&4&2&3\\201&2&2&5\\120&5&1&3\\120&3&1&5\\102&1&1&7\\030&6&0&3\\021&4&0&5\\012&2&0&7\\003&0&0&9\end{pmatrix}.$$

$$\begin{bmatrix}V_k&v_k\end{bmatrix}=V_{k+1}\\\begin{bmatrix}Z_k&z_k\end{bmatrix}=Z_{k+1}=\begin{bmatrix}S_k-H_0Y_k&s_k-H_0y_k\end{bmatrix}=S_{k+1}-H_0Y_{k+1}\\\begin{bmatrix}(M_k)_{11}&0_{k\times1}\\0_{1\times k}&0\end{bmatrix}=\begin{bmatrix}0_{k\times k}&0_{k\times1}\\0_{1\times k}&0\end{bmatrix}=0_{(k+1)\times(k+1)}\\\begin{bmatrix}(M_k)_{21}&0_{k\times1}\\\frac{y_k^TV_k}{\rho_k}\end{bmatrix}=\begin{bmatrix}(R_k^{YY})^T&\\y_k^TV_k&\frac{y_k^Tv_k}{\rho_k}\end{bmatrix}=(R_{$$