

Dany Cajas

Advanced Portfolio Optimization

A Cutting-edge Quantitative Approach

 Springer

Dany Cajas

Advanced Portfolio Optimization

a Cutting-edge Quantitative Approach

Apr 2025

Springer Nature

Contents

1	Introduction	1
1.1	Origin, Audience and Style of this Book	1
1.2	History of Portfolio Optimization	2
1.3	Structure of the Book	5
2	Why use Python?	9
2.1	Advantages of Python	9
2.2	How to Install Python?	9
2.3	How to Work with Python?	11
Part I Parameter Estimation		
3	Sample Based Methods	15
3.1	Moments	16
3.1.1	Raw Moments	16
3.1.2	Central Moments	16
3.1.3	Standardized Central Moments	17
3.1.4	Covariance Matrices	18
3.1.5	Linear Moments or L-Moments	23
3.2	Exponential Weighted Moving Average (EWMA)	26
3.3	Multivariate Time Series	29
3.3.1	Vector Autoregressive Models	29
3.3.2	Vector Moving Average Models	31
3.3.3	Vector Autoregressive Moving Average Models	32
3.4	Shrinkage Estimators	35
3.4.1	Shrinkage Mean	35
3.4.2	Shrinkage Covariance	39
3.5	Random Matrix Theory	42
3.5.1	Marchenko-Pastur Distribution	42
3.5.2	Denoising	44
3.5.3	Detrending	47
3.6	Graph Based	48

3.6.1	Graphical Lasso	48
3.6.2	JLogo Covariance	49
4	Risk Factors Models	53
4.1	General Risk Factors Models	53
4.2	Explicit Factors Models	56
4.2.1	Fama-French Model	56
4.2.2	Key Rate Durations and Convexities for Bonds	59
4.3	Implicit Factor Models	62
4.3.1	Principal Components Regression (PCR)	62
5	Black Litterman Models	67
5.1	Black Litterman Model	68
5.1.1	General Equilibrium Model	68
5.1.2	Reference Model	68
5.1.3	View Creation Model	69
5.1.4	Theil and Goldberg Mixed Estimation Model	69
5.1.5	Black Litterman Model	70
5.2	Augmented Black Litterman	74
5.3	Black Litterman Bayes	77
6	Codependence and Dissimilarity Measures	83
6.1	Codependence or Similarity Measures	84
6.1.1	Pearson Correlation	84
6.1.2	Spearman Rank Correlation	85
6.1.3	Kendall Tau	87
6.1.4	Gerber Statistic	88
6.1.5	Distance Correlation	92
6.1.6	Mutual Information	94
6.1.7	Tail Dependence Coefficient	97
6.2	Dissimilarity and Distance Measures	99
6.2.1	Codependence Based Distance	99
6.2.2	Variation of Information	101
6.2.3	Tail Dissimilarity	101
Part II Convex Portfolio Optimization		
7	Convex Risk Measures	105
7.1	Classification Based on Properties	106
7.1.1	Coherent Risk Measures	106
7.1.2	Distortion Risk Measures	107
7.1.3	Spectral Risk Measures	108
7.1.4	φ -Divergence Risk Measures	108
7.1.5	Ordered Weighted Average Risk Measures	109
7.2	Classification Based on the Characteristic It Quantifies	115
7.2.1	Deviation Risk Measures	116
7.2.2	Downside Risk Measures	132

7.2.3	Range Risk Measures	152
7.2.4	Drawdown Risk Measures	162
7.2.5	Higher Moments Risk Measures	175
8	Return-Risk Trade-Off Optimization	195
8.1	Arithmetic and Geometric Returns	196
8.1.1	Arithmetic Returns	196
8.1.2	Geometric Returns or Kelly Criterion	197
8.2	Classic Objective Functions	201
8.2.1	Minimization of Risk	201
8.2.2	Maximization of Return	207
8.2.3	Maximization of an Utility Function	212
8.2.4	Maximization of a Risk Adjusted Return Ratio	216
9	Real Features Constraints	221
9.1	Linear Constraints	222
9.2	Index Tracking	236
9.3	Other Convex Features	242
9.4	Mixed Integer Constraints	247
10	Risk Parity Optimization	259
10.1	Risk Parity with Assets	260
10.1.1	Risk Contribution and Euler Decomposition	260
10.1.2	Least Squares Approach	262
10.1.3	Risk Budgeting Approach	266
10.1.4	Mixed Integer Programming (MIP) Approach	269
10.1.5	Extensions to Risk Parity with Assets	272
10.2	Risk Parity with Risk Factors	275
10.2.1	Risk Contribution and Euler Decomposition of Risk Factors	275
10.2.2	Risk Budgeting Approach with Risk Factors	278
10.2.3	Extensions to Risk Parity with Risk Factors	282
11	Robust Optimization	287
11.1	Resampling	288
11.2	Stochastic Optimization	294
11.3	Worst Case Optimization	299
11.3.1	Box Uncertainty Sets	300
11.3.2	Elliptical Uncertainty Sets	305
11.4	Near Optimal Centering	310
11.4.1	Analytic Center of a Set of Inequalities	310
11.4.2	NOC Portfolio for Convex Risk Measures	311

Part III Machine Learning Portfolio Optimization

12 Hierarchical Clustering Portfolios	321
12.1 Hierarchical Risk Parity	322
12.1.1 Hierarchical Clustering	322
12.1.2 Seriation or Quasi-Diagonalization	329
12.1.3 Recursive Bisection	330
12.1.4 Conclusions	332
12.2 Hierarchical Equal Risk Contribution	333
12.2.1 Optimal Number of Clusters	333
12.2.2 Hierarchical Recursive Bisection	336
12.2.3 Conclusions	338
12.3 Nested Clustered Optimization	339
12.3.1 Intra-Cluster Asset Allocation	340
12.3.2 Inter-Cluster Asset Allocation	341
12.3.3 Conclusions	342
13 Graph Theory Based Portfolios	345
13.1 Constraints on Graphs	346
13.1.1 Graphical Representation of Graphs	346
13.1.2 Matrix Representation of Graphs	347
13.1.3 Centrality Measures of Graphs	349
13.1.4 Graphs in Financial Markets	352
13.1.5 Graph Measures for Portfolios	354
13.1.6 Centrality Measure Constraint	356
13.1.7 Neighborhood Constraint	357
13.2 Clusters Constraint	364
13.2.1 Graphical Representations of Dendrograms	364
13.2.2 Matrix Representation of Dendrograms	365
13.2.3 Dendrogram Measures for Portfolios	367
13.2.4 Clusters Constraints	367
Part IV Backtesting	
14 Generation of Synthetic Data	377
14.1 Block Bootstrap	378
14.1.1 Moving Bootstrap	378
14.1.2 Circular Bootstrap	379
14.1.3 Stationary Bootstrap	380
14.1.4 Optimal Block Length	381
14.2 Copulas	382
14.2.1 Definition	383
14.2.2 Select the Best Distribution per Asset	383
14.2.3 Elliptical Copulas	384
14.2.4 Archimedean Copulas	389
14.2.5 Independent Copula	395
14.2.6 Empirical Copula	397
14.2.7 Gaussian Mixture Copula	400

14.3	Econometric Models	402
14.3.1	Time Series Models Based Scenarios	402
14.3.2	Risk Factors Models Based Scenarios	405
14.4	Conclusions	410
15	Backtesting Process	411
15.1	The Walk-Forward Method	412
15.2	The Cross-Validation Method	418
15.3	The Combinatorial Purged Cross-Validation Method	425
 Part V Appendix		
A	Linear Algebra	439
A.1	Matrix Operations	439
A.2	Special Matrices	443
B	Convex Optimization	445
B.1	Linear Programming	445
B.2	Quadratic Programming	446
B.3	Second Order Cone Programming	447
B.4	Semidefinite Programming	447
B.5	Exponential Cone Programming	448
B.6	Power Cone Programming	450
B.7	Convex Fractional Programming	451
B.8	Conic Duality	452
C	Mixed Integer Programming	455
C.1	Mixed Integer Convex Programming	455
C.2	Mixed Integer Fractional Programming	457
	References	459