Sparse Hemispherical Arrays with Hierarchical Low Discrepancy Sequence Sampling

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***Abstract*— Sparse spherical arrays have grating lobes due to large element spacings. Aperiodic spacing can reduce the grating lobes, however random approaches typically increase the average sidelobe level. This paper explores the use of a deterministic hierarchical random-like distribution based on the low-discrepancy Van der Corput sequence. Array pattern results are presented for hemispherical arrays with element spacings larger than one wavelength and we show that this approach removes the grating lobes while allowing one to add or remove elements without needing to recalculate positions.**

***Index Terms*—aperiodic array, conformal array, hemispherical array, phased array, sparse array.**

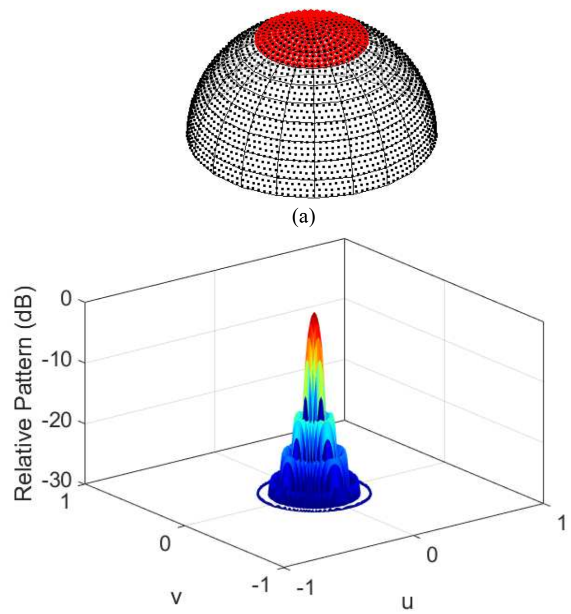
I. Introduction

Spherical arrays have several advantages over planar technologies when scanning the main beam, such as low polarization loss, low gain loss as a function of scan angle, and beam pointing that is independent of frequency [1]-[4]. However, these arrays are usually large and have many elements which makes them expensive. Typically, designers try to minimize the number of elements on the aperture, but the increased element spacing produces grating lobes that degrade the array performance. For a dense array, thinning and aperiodic spacing are used to lower sidelobe levels [4]-[6], but for sparse arrays with element spacings larger than one wavelength, low sidelobes are not an option due to the grating lobes. In this work we present the use of a hierarchical deterministic random-like distribution [7]-[9], namely Halton sampling, for sparse hemispherical arrays. We show that the proposed element distributions eliminate the grating lobes while using only about 22% of the elements in a fully populated array.

II. Element Sampling on a Hemispherical Surface

A. Concentric Ring

Hemispherical arrays with elements equally distributed around concentric rings yields a symmetrical element lattice when the beam points at nadir. While other, such as pseudo-rectangular or pseudo-triangular can be used, have more uniform lattices compared to concentric rings. As an example, a hemispherical array of isotropic elements with a sphere diameter of 10λ is shown in Fig. 1 (a) **Need to show and label axes on figure**. The element spacing in theta and phi are set to 0.5λ, so the full array has 2509 elements. In these hemispherical arrays, not all the elements are active at once, and typically an active region is selected to scan the beam in the desired direction. In the studies reported in this work, we consider a beam at nadir and an active region of about 25 degrees in theta, which corresponds to 9 concentric rings at the top with this element spacing. The radiation pattern of this array appears in Fig. 1 (a). It can be seen in Fig. 1 (b) that with this element spacing produces low sidelobe levels at a cost of a large number of elements.



(a)

(b)

Fig. 1. (a) Concentric ring element placing on the hemispherical array with 0.5λ spacing in theta and phi. (b) Array pattern.

B. Hierarchical Low-Discrepancy Sampling Sequence

Sparse arrays use fewer number of elements compared to dense arrays, however, the large element spacing results in grating lobes that degrade the array performance. Random element grids are typically used approaches can be used to remove the periodicity but have multiple problems [4]. Here we explore a different approach that uses a hierarchical deterministic random-like distribution. The hierarchical property allows one to add or remove elements without the need to recalculate element locations. Our approach is based on the Van der Corput sequence, and a hierarchical distribution to a two-dimensional grid by Halton sampling. The sequence is given by

$$\Psi_b(n) = \sum_{k=0}^{L-1} a_k b^{-k-1}\,, \tag{1}$$

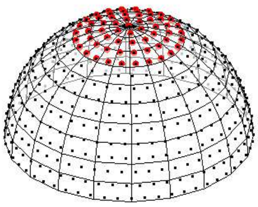
where, b is the prime base in which number n is represented and a_k is an integer in $[0,\, b-1]$. Halton sampling is given by

$$\left(x_n, y_n\right) = \left(\Psi_{b_1}(n), \Psi_{b_2}(n)\right),\quad n = 0,1,2,\ldots,N-1, \tag{2}$$

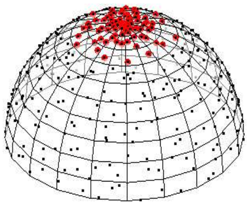
where b_1 and b_2 are two different prime bases. A cosinus mapping is then used to project these points onto the hemisphere [3].

III. Sparse Hemispherical Arrays

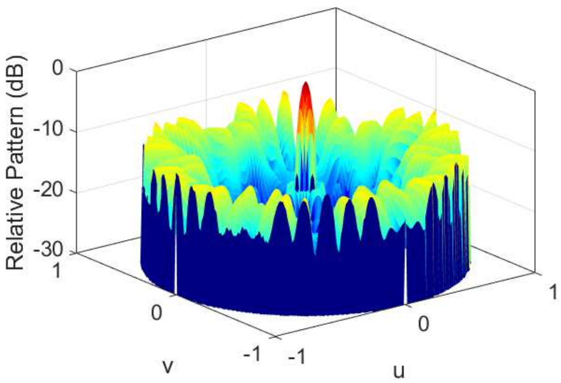
We consider two examples of sparse hemispherical arrays using the proposed approach. The dense array in section II had 274 active elements. We reduce the number of active elements to 90, which corresponds to an element spacing of 1.1λ for the concentric ring array. The Halton sampling uses the same number of elements in the active region. The array geometries and the radiation patterns are given in Fig. 2. With uniform sampling grating lobes are appearing in the visible region, and the peak sidelobe level (PSLL) is about -9.97 dB. On the other hand, the Halton sampling shows a much-improved radiation pattern with no grating lobe, and a PSLL of about -13.76 dB. Increasing the sparsity level further, we only use 61 elements in the active region. Due to the hierarchical property, there is no need to recalculate the Halton points, and the last 29 points are just removed. The array geometries along with the radiation patterns are given in Fig. 3, where we note that again the Halton sampling can remove the grating lobes. The PSLL for the uniform and Halton samplings are -5.36 and -11.08 dB, respectively.



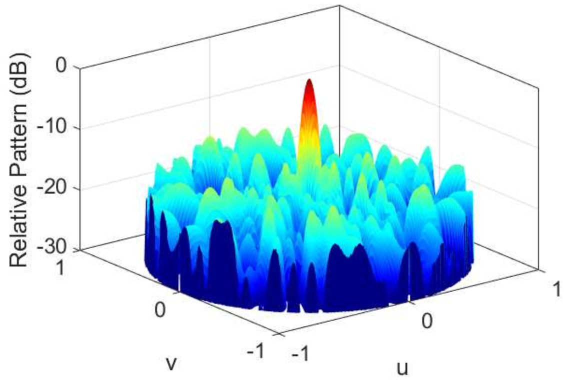
(a)



(b)

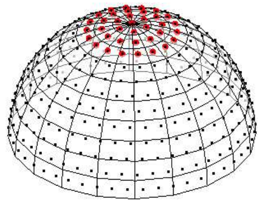


(c)

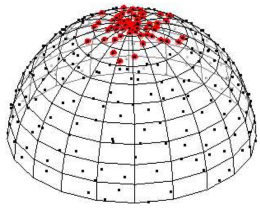


(d)

Fig. 2. Element placing on the hemispherical array: (a) concentric ring, (b) Halton sampling. Array pattern with concentric ring (c), and Halton (d), distributions. For the concentric ring, the element spacing in theta and phi is 1.1λ.



(a)



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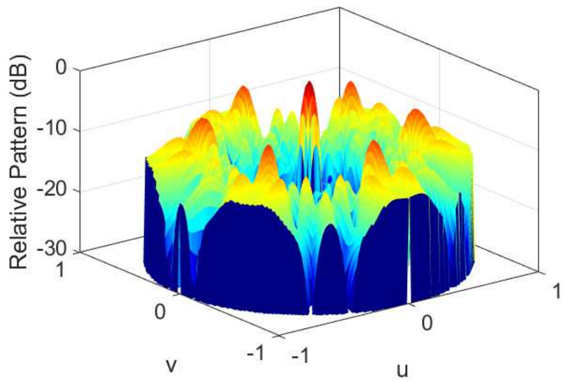

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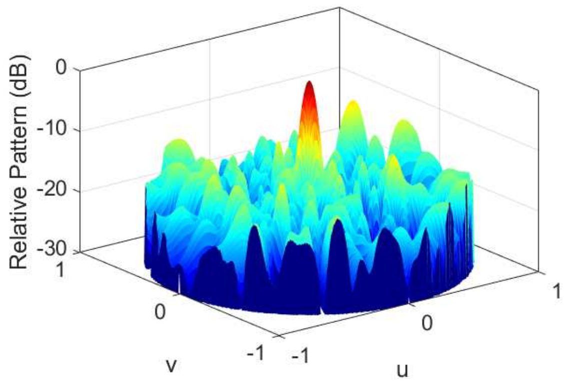

(d)

Fig. 3. Element placing on the hemispherical array: (a) concentric ring, (b) Halton sampling. Array pattern with concentric ring (c), and Halton (d), distributions. For the concentric ring, the element spacing in theta and phi is 1.24λ.

IV. CONCLUSIONS

This paper explores the use of a deterministic hierarchical low-discrepancy sampling for sparse hemispherical arrays. We compare the proposed approach to concentric ring arrays, and we show that the proposed element distributions can remove the grating lobes while using only about 22% of the elements of a dense array.

REFERENCES

[1] L. Josefsson and P. Persson, *Conformal Array Antenna Theory and Design*, IEEE Press, Wiley-Interscience: Hoboken, NJ, 2006.

[2] Boris Tomasic, John Turtle, and L. Shiang, "Spherical Arrays - Design Considerations," 18th International Conference on Applied Electromagnetics and Communications, 2005, pp. 1-8.

[3] R. L. Haupt, “A sparse hammersley element distribution on a spherical antenna array for hemispherical radar coverage,” *2017 IEEE Radar Conference (RadarConf)*, 2017, pp. 1383-1385.

[4] R. L. Haupt, *Antenna Arrays: A Computational Approach*, Hoboken, NJ: Wiley, 2010.

[5] R. Willey, “Space tapering of linear and planar arrays,” *IEEE Trans. Antennas Propag.*, vol. 10, no. 4, pp. 369-377, Jul. 1962.

[6] M. Skolnik, J. Sherman, III, and F. Ogg, Jr., “Statistically designed density-tapered arrays,” *IEEE Trans. Antennas Propag.*, vol. 12, no. 4, pp. 408-417, Jul. 1964.

[7] J. G. van der Corput, Verteilungsfunktionen I, *Akademie van Wetenschappen*, vol. 38, 1935, pp. 813-821.

[8] T. T. Wong, W. S. Luk, and P. A. Heng, “Sampling with Hammersley and Halton points,” *J. Graphics Tools*, vol. 2, no. 2, pp 9-24, 1997.

[9] T. Torres, N. Anselmi, P. Nayeri, P. Rocca, and R. L. Haupt, “Low discrepancy sparse phased array antennas,” *Sensors* 2021, 21(23), 7816, pp. 1-20, Nov. 2021.