$$s\begin{bmatrix} a \\ b \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \leftarrow \text{coordinates relative to camera focus}$$

Let w = object z coordinate rel to camera focus

Let $s = z/w = 1$

$$\underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{\underline{a}} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \end{bmatrix} \begin{bmatrix} x/w \\ y/w \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & 0 \\ 0 & f_y \end{bmatrix}}_{F} \underbrace{\begin{bmatrix} x/w \\ y/w \end{bmatrix}}_{\underline{x}} + \underbrace{\begin{bmatrix} c_x \\ c_y \end{bmatrix}}_{\underline{c}}$$

image coordinates

$$\underline{a} = F\underline{x} + \underline{c} \qquad F^{-1} = \begin{bmatrix} 1/f_x & 0 \\ 0 & 1/f_y \end{bmatrix}$$

$$F\underline{x} = \underline{a} - \underline{c}$$

$$\underline{x} = F^{-1}\underline{a} - F^{-1}\underline{c}$$

$$\begin{bmatrix} x/w \\ y/w \end{bmatrix} = \begin{bmatrix} 1/f_x & \\ & 1/f_y \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} - \begin{bmatrix} c_x/f_x \\ c_y/f_y \end{bmatrix}$$

$$\frac{d(x/w)}{du} = \frac{1}{f_x}$$

$$\frac{d(y/w)}{dv} = \frac{1}{f_y}$$

$$\text{rayvecfactor} = \frac{1}{\sqrt{(x/w)^2 + (y/w)^2 + 1}} = \frac{1}{\sqrt{\left(\frac{a-c_x}{f_x}\right)^2 + \left(\frac{b-c_y}{f_y}\right)^2 + 1}}$$

$$\text{rayvec} = \begin{bmatrix} x/w \\ y/w \\ 1 \end{bmatrix} * \text{rayvecfactor}$$

(where $\frac{x}{w}$ and $\frac{y}{w}$ are #'s determined by which pixel (a,b))

or in projective (4D) coords

$$\frac{d\,\text{rayvecfactor}}{da} = -\frac{1}{2}\,\text{rayvecfactor}^3\, 2\,\frac{a-c_x}{f_x^2}$$

$$= \frac{-\text{rayvecfactor}^3 (a-c_x)}{f_x}$$

$$\text{rayvec} = \begin{bmatrix} x/w \\ y/w \\ 1 \\ 0 \end{bmatrix} * \text{rayvecfactor}$$

$$\frac{d\,\text{rayvec}}{da} = \begin{bmatrix} 1/f_x \\ 0 \\ 0 \\ 0 \end{bmatrix} * \text{rayvecfactor} + \begin{bmatrix} x/w \\ y/w \\ 1 \\ 0 \end{bmatrix} * \frac{d\,\text{rayvecfactor}}{da}$$

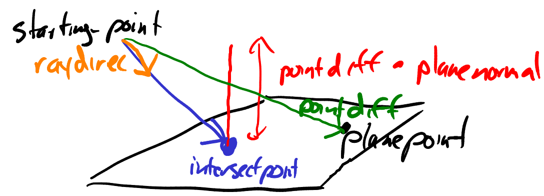
(this is the old horiz_ray_direc_shift once converted to object coordinates)

in object coords:

$$rayvecobj = A \, rayvec$$

$$\frac{d\,rayvecobj}{da} = A \begin{bmatrix} 1/fx \\ 0 \\ 0 \\ 0 \end{bmatrix} * rayvecfactor + A \begin{bmatrix} x/w \\ y/w \\ 1 \\ 0 \end{bmatrix} * \frac{d\,rayvecfactor}{da}$$

$\frac{d\,rayvecobj}{db}$ can be defined similarly



If this ray intersects a plane, we can evaluate how the intersection point shifts with a and b

$$pointdiff = planepoint - starting\_point$$

$$intersectpoint = startingpoint + \frac{pointdiff \cdot planenormal}{raydirec \cdot planenormal} * raydirec$$

$$d\,intersectpoint = \frac{(raydirec \cdot planenormal)(pointdiff \cdot planenormal)\, draydirec - (pointdiff \cdot planenormal)\, raydirec\, (planenormal \cdot draydirec)}{(raydirec \cdot planenormal)^2}$$

$$d\,intersectpoint = \frac{pointdiff \cdot planenormal}{(raydirec \cdot planenormal)^2} \left[(raydirec \cdot planenormal)\, draydirec - raydirec\,(planenormal \cdot draydirec) \right]$$

where draydirec can be in any coord frame with respect to changes da or db

The above is calculated by ray_to_plane_raydirec_shift()