



Ultimate GemCutter: Commutativity in Concurrent Program Verification

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Commutativity Simplifies Proofs of Concurrent Programs

Concurrent Program

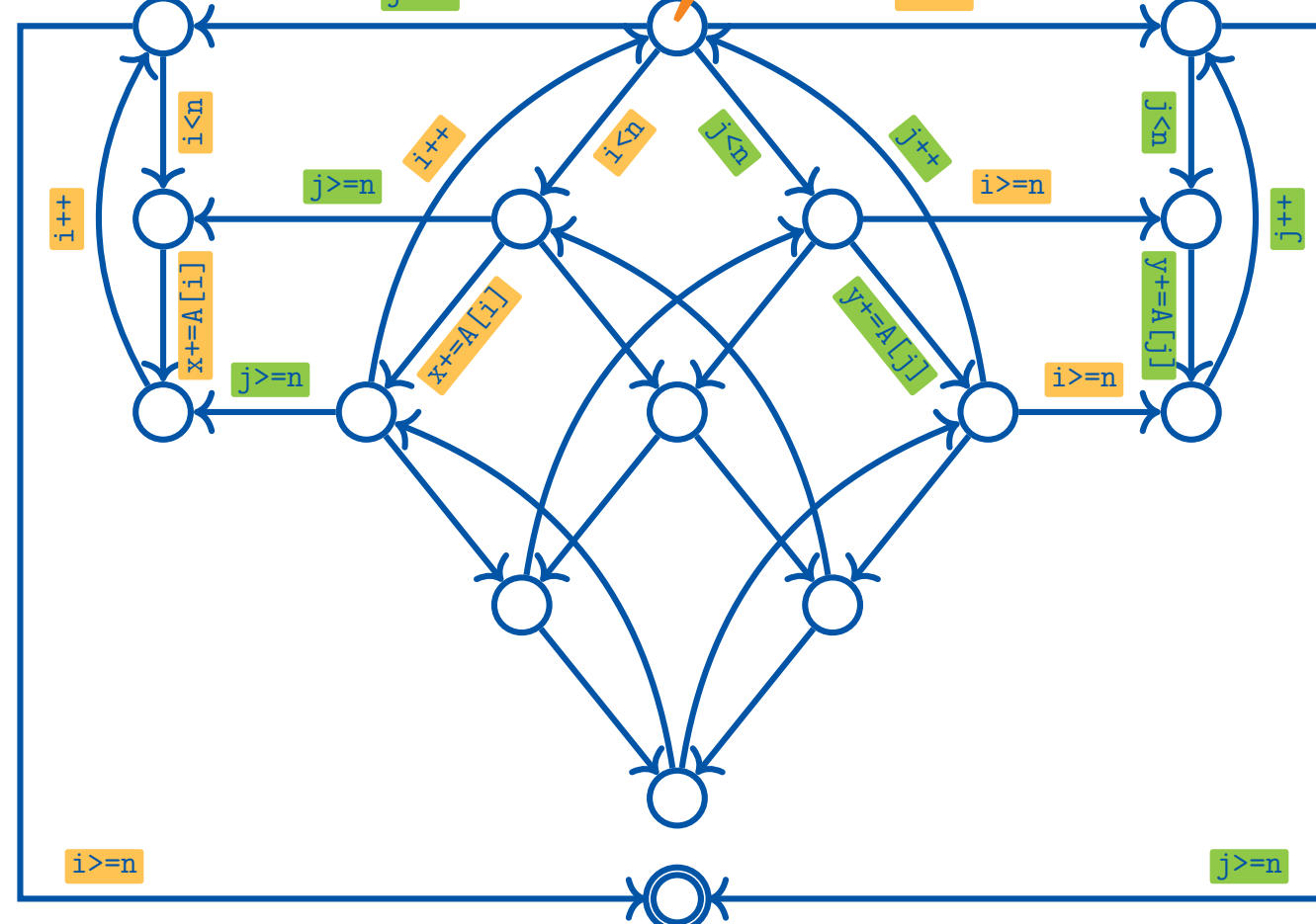
$$\{x = y = i = j = 0\}$$

```
while (i < n) {
  x += A[i];
  i++;
}
||
while (j < n) {
  y += A[j];
  j++;
}
```

$$\{x = y\}$$

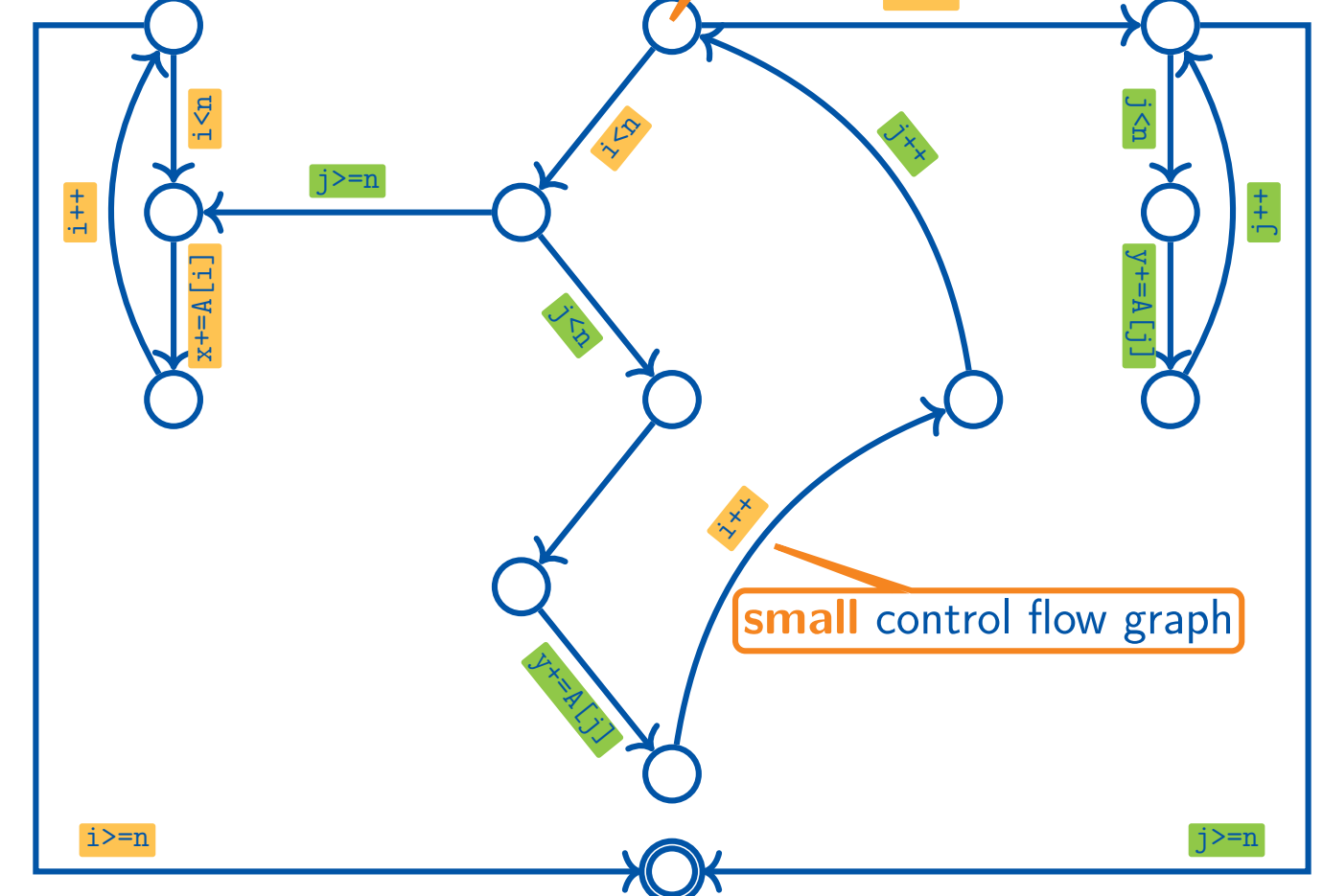
All Interleavings

complex invariant: $x = \sum_{k=0}^i A[k] \wedge y = \sum_{k=0}^j A[k] \wedge i \leq n \wedge j \leq n$



A Sound Reduction

simple invariant: $x = y \wedge i = j$



Commutativity

Many pairs of statements **commute**:

i.e., order of execution does not matter

Example: $x += A[i] \quad y += A[j] \sim y += A[j] \quad x += A[i]$

Extension: **proof-sensitive commutativity**

Example: $*x = 0 \quad *y = 1 \sim *y = 1 \quad *x = 0$
if we have proven that $x \neq y$

swapping adjacent commuting statements

\rightsquigarrow **equivalent** traces

Reduction

representative subset of program traces: at least one representative per equivalence class

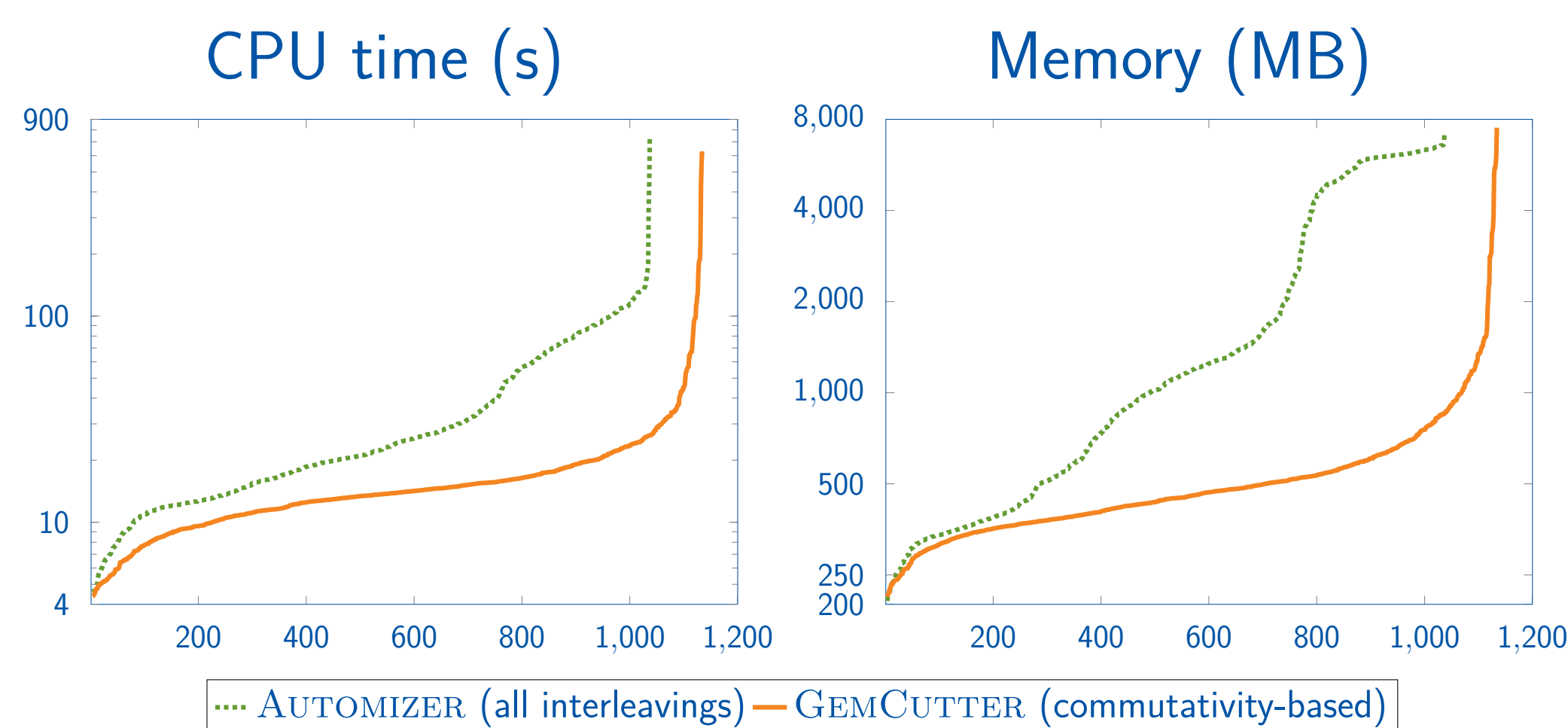
Soundness:

one trace correct \Rightarrow all equivalent traces correct

correctness of reduction \Rightarrow correctness of program

Performance

Evaluation shows significant advantages over a state-of-the-art verifier (Ultimate Automizer):



Competitions:

- **SV-COMP'24:** 2nd place in *ConcurrencySafety*
- **SV-COMP'23:** 3rd place in *ConcurrencySafety*
- **SV-COMP'22:** 3rd place in *ConcurrencySafety*, 1st place in *NoDataRace (demo)*

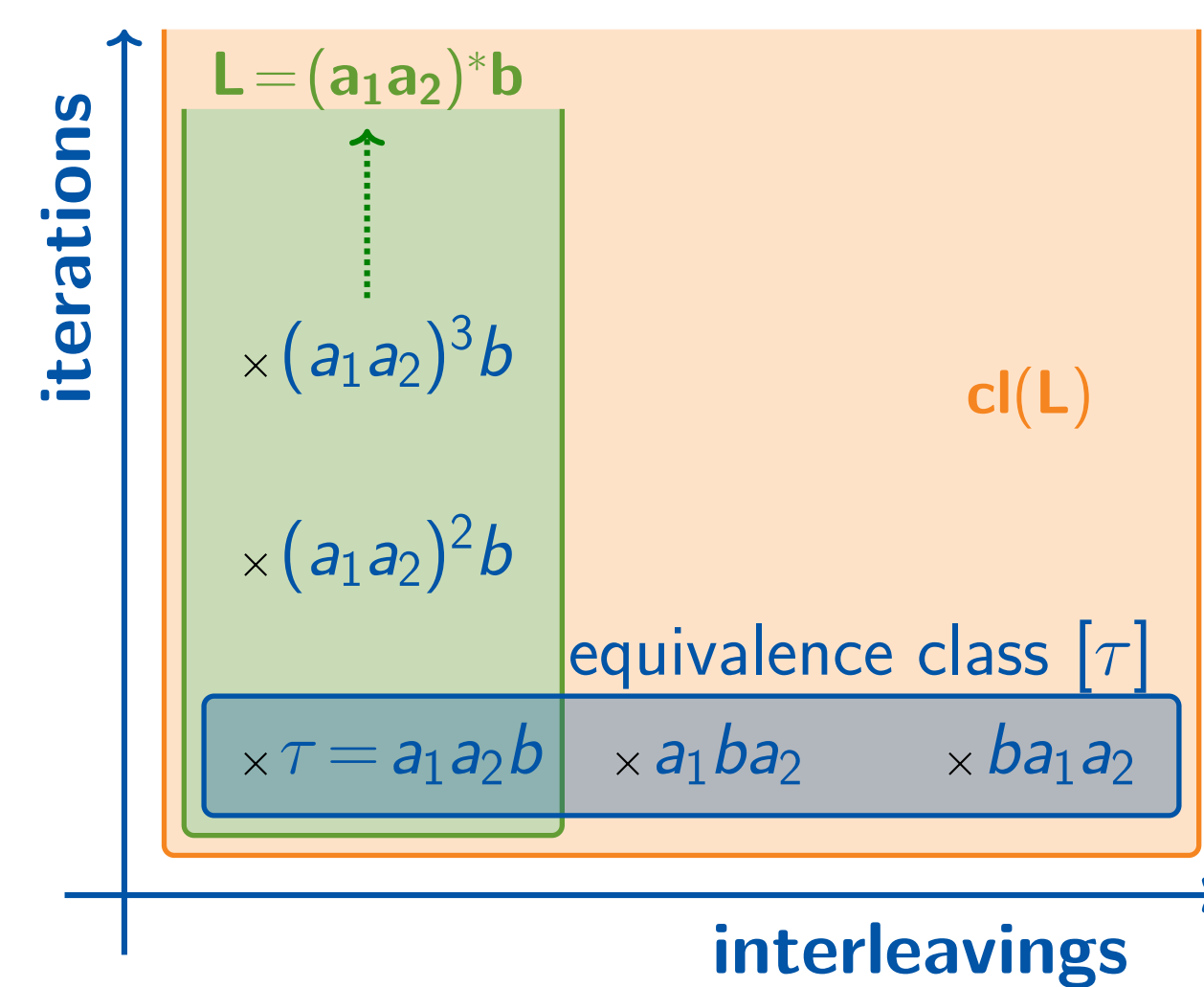
Verification Principle

GemCutter **generalizes** from spurious counterexamples τ to larger sets of correct traces:

trace abstraction

generalizes across loop iterations to a set of traces L

commutativity allows for generalization across interleavings to the set $cl(L)$ of all equivalent traces



If $cl(L)$ contains all program traces, the program is correct.

Equivalently: If L contains all traces of a reduction, then the program is correct.

Commutativity & Verification

choice of representatives affects proof simplicity

► **challenge:** select suitable representatives

choice of proof affects possible commutativity

► **challenge:** find useful *abstract* commutativity

partial order reduction algorithms speed up verification

► **challenge:** adapt classical POR algorithms

commutativity reasoning is widely applicable

► **challenge:** extend to more programs & properties

- [SV-COMP'22] *Ultimate GemCutter and the Axes of Generalization*, Klumpp, Dietsch, Heizmann, Schüssele, Ebbinghaus, Farzan and Podelski, 2022
- [PLDI'22] *Sound Sequentialization for Concurrent Program Verification*, Farzan, Klumpp and Podelski, 2022
- [POPL'23] *Stratified Commutativity in Verification Algorithms for Concurrent Programs*, Farzan, Klumpp and Podelski, 2023
- [POPL'24] *Commutativity Simplifies Proofs of Parameterized Programs*, Farzan, Klumpp and Podelski, 2024