Fitting for the PSF in an image

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1 Selecting Image Pixels for the Fit

The goal is to select a set of pixels the values of which will be used to fit for the PSF in an image. We would like to select a set of pixels belonging to isolated sources and which contain significant amount of light from the source they are assigned to.

Some definitions:

m_p The measured electron counts of pixel p

(x_n, y_n) The position of the n-th source on the image

B_n, b_n The background and its standard variance as estimated for the n-th source.

α A tunable parameter setting how far above the standard deviation a pixel value should be in order for the pixel to be included in the fit for a source PSF.

P A tunable parameter selecting how many pixel at minimum a source must include in order to be included in the PSF fit.

We will assume that we start with a list of source positions (x_n, y_n), backgrounds (B_n) and background variances (b_n). Fit pixels are then selected following these steps for each source:

1. Starting from the pixel which contains the source center, move in the positive and negative y direction until the condition $m_p^2 > B_n + \alpha (m_p + b_n)$ is no longer satisfied, adding all pixels to the list of pixels belonging to source n.
2. For each of the previously selected pixels perform the same procedure, but moving in the positive and negative x directions.
3. If a pixel is included in the pixel lists for more than one source eliminate all sources containing that pixel from the fit.
4. Eliminate bad pixels
5. Mark saturated pixels
6. Eliminate sources for which the number of pixels is less than P

2 Fitting a Fixed PSF Model

We assume that some model containing some (small) number of free parameters is available for individual source background subtracted PSFs which only depends on the position of the source on the image, and that changing the brightness of a source only changes the overall scaling of the PSF.

Given some values for the model parameters, the quantity to minimize (R) is calculated using the following steps:

1. For each source (n), using local polynomial approximation to each PSF we calculate the integral over each selected pixel (f_{ni}) to a fractional precision of $0.01(m_p + b_n)/(m_p - B_n)$.

2. The amplitude for the model PSF of each source is estimated as:

$$A_n \equiv \frac{\sum_p \frac{(m_p - B_n) f_{np}}{m_p + b_n}}{\sum \frac{f_{np}^2}{m_p + b_n}}$$

3. For each saturated pixel where $A_n f_{np}$ is more than m_p, replace m_p with $A_n f_{np}$.

4.

$$R \equiv \sum_n \sum_p \frac{\left(m_p - B_n - A_n f_{np}\right)^2}{m_p + b_n}$$

Two modes of PSF fitting are supported:

Source by source: the PSF parameters for each source are derived independently

Ensemble: the PSF parameters are assumed to vary smoothly accross the image. A low order polynomial is used to represent each parameter and the fit is done on the coefficients of all polynomials such that the combined residuals from all source pixels are minimized simultaneously. Fitting is initially done with constant PSF parameters, once the best fit for those is found, linear terms are added and so on, until the prescribed polynomial order is reached.

The minimization is done by the Newton-Raphson method, using analytic first and second derivatives of the residual with respect to the source parameters in source by source mode or the polynomial expansion coefficients is ensemble mode. This means that a nearby local minimum of the residuals is found, so a reasonable initial guess for the PSF parameters is of some importance.

3 Calculating Flux

In order to calculate the flux from amplitude and PSF parameters:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left\{-\frac{1}{2}\left[S(x^2+y^2)+D(x^2-y^2)+2Kxy\right]\right\} dxdy = \frac{2\pi A}{\sqrt{S^2-D^2-K^2}} \tag{1}$$

Under the conditinos that $S^2 > D^2 + K^2$ and $S + D > 0$.