A Technical Appendix

A.1 Proofs for Section 4

To prove the results from this section, it is convenient to make the following additional observation about the relation between $\mathcal{N}$ and Δ in our procedure:

Proposition 1. *At the start of each iteration of the loop in Procedure 2, it holds that* $(\mathcal{D}' \Cup \mathcal{N}) = (\mathcal{D}' \Cup \Delta)$.

Proof. Initially, $\mathcal{N} = \emptyset$ and $\Delta = \mathcal{D}' = \mathcal{D}$, so the statement of the proposition holds at the start of the first iteration. We next consider further iterations.

To show that $(\mathcal{D}' \Cup \mathcal{N}) \subseteq (\mathcal{D}' \Cup \Delta)$, we assume that $\mathcal{D}' \Cup \mathcal{N} \models M@t$, for some relational fact $M@t$. Therefore, we have either $\mathcal{D}' \models M@t$, or $\mathcal{D}' \not\models M@t$ and $\mathcal{N} \models M@t$. If $\mathcal{D}' \models M@t$, then clearly $\mathcal{D}' \Cup \Delta \models M@t$. Next, consider the case when $\mathcal{D}' \not\models M@t$ and $\mathcal{N} \models M@t$. Since $\mathcal{N} \models M@t$, we have also $\mathcal{D}' \Cup \mathcal{N} \models M@t$. Thus, there need to exist intervals ϱ' and ϱ'', both of which include t, and such that $M@\varrho' \in \mathcal{N}$, $M@\varrho'' \in \mathcal{D}' \Cup \mathcal{N}$, and $\varrho' \subseteq \varrho''$. Hence, $M@\varrho'' \models M@\varrho'$, and so, by the definition of Δ in Line 5, we obtain that $M@\varrho'' \in \Delta$. Therefore $\Delta \models M@t$, and thus, $\mathcal{D}' \Cup \Delta \models M@t$.

To show that $(\mathcal{D}' \Cup \Delta) \subseteq (\mathcal{D}' \Cup \mathcal{N})$, we assume that $\mathcal{D}' \Cup \Delta \models M@t$, for some relational fact $M@t$. Therefore, we have $\mathcal{D}' \models M@t$ or $\Delta \models M@t$. If $\mathcal{D}' \models M@t$, then $\mathcal{D}' \Cup \mathcal{N} \models M@t$. Otherwise, $\Delta \models M@t$, and so, there needs to exist ϱ such that $t \in \varrho$ and $M@\varrho \in \Delta$. By the definition of Δ in Line 5, we have $\Delta \subseteq \mathcal{D}' \Cup \mathcal{N}$, so $M@\varrho \in (\mathcal{D}' \Cup \mathcal{N})$, and therefore $\mathcal{D}' \Cup \mathcal{N} \models M@t$.

Theorem 1 (Soundness): Consider Procedure 2 running on input Π and $\mathcal{D}$. Upon the completion of the kth (for some $k \in \mathbb{N}$) iteration of the loop of Procedure 2, it holds that $\mathfrak{I}_{\mathcal{D}'} \subseteq T_{\Pi}^{k}(\mathfrak{I}_{\mathcal{D}})$.

Proof. For each $k \in \mathbb{N}$, we let $\mathcal{N}_k$, Δ_k, and $\mathcal{D}_k$ denote the contents of, respectively, $\mathcal{N}$, Δ, and $\mathcal{D}'$ in Procedure 2 upon the completion of the kth iteration of the loop. Thus, it suffices to show, inductively on $k \in \mathbb{N}$, that $\mathfrak{I}_{\mathcal{D}_k} \subseteq T_{\Pi}^{k}(\mathfrak{I}_{\mathcal{D}})$.

In the base case, by the initialisation of the procedure, we have $\mathcal{D}_0 = \mathcal{D}$. Moreover, $T_{\Pi}^{0}(\mathfrak{I}_{\mathcal{D}}) = \mathfrak{I}_{\mathcal{D}}$, and so, $\mathfrak{I}_{\mathcal{D}_0} \subseteq T_{\Pi}^{0}(\mathfrak{I}_{\mathcal{D}})$, as required.

For the inductive step, we assume that $\mathfrak{I}_{\mathcal{D}_k} \subseteq T_{\Pi}^{k}(\mathfrak{I}_{\mathcal{D}})$, for some $k \in \mathbb{N}$, and that the procedure enters the $k+1$st iteration of the loop. If the $k+1$st iteration of the loop breaks in Line 6, then $\mathcal{D}_{k+1} = \mathcal{D}_k$. By the inductive assumption we have $\mathfrak{I}_{\mathcal{D}_k} \subseteq T_{\Pi}^{k}(\mathfrak{I}_{\mathcal{D}})$, so $\mathfrak{I}_{\mathcal{D}_{k+1}} \subseteq T_{\Pi}^{k}(\mathfrak{I}_{\mathcal{D}})$, and thus, $\mathfrak{I}_{\mathcal{D}_{k+1}} \subseteq T_{\Pi}^{k+1}(\mathfrak{I}_{\mathcal{D}})$.

Now, assume that the $k+1$ iteration of the loop does not break in Line 6. To show that $\mathfrak{I}_{\mathcal{D}_{k+1}} \subseteq T_{\Pi}^{k+1}(\mathfrak{I}_{\mathcal{D}})$, we assume that $\mathfrak{I}_{\mathcal{D}_k+1} \models M@t$, for some relational fact $M@t$. By Line 5, we obtain that $\mathcal{D}_{k+1} = \mathcal{D}_k \Cup \mathcal{N}_{k+1}$. Therefore, we have $\mathcal{D}_k \models M@t$ or $\mathcal{N}_{k+1} \models M@t$.

Case 1: $\mathcal{D}_k \models M@t$. By the inductive assumption, we know that $\mathfrak{I}_{\mathcal{D}_k} \subseteq T_{\Pi}^{k}(\mathfrak{I}_{\mathcal{D}})$, so $T_{\Pi}^{k}(\mathfrak{I}_{\mathcal{D}}) \models M@t$. Clearly, $T_{\Pi}^{k}(\mathfrak{I}_{\mathcal{D}}) \subseteq T_{\Pi}^{k+1}(\mathfrak{I}_{\mathcal{D}})$, so $T_{\Pi}^{k+1}(\mathfrak{I}_{\mathcal{D}}) \models M@t$.

Case 2: $\mathcal{N}_{k+1} \models M@t$. By Line 3, we have $\mathcal{N}_{k+1} = \Pi[\mathcal{D}_k \vdots \Delta_k]$. Thus, by Definition 3, there is a rule $r \in \Pi$, say of the form $M' \leftarrow M_1 \wedge \cdots \wedge M_n$, such that $\mathsf{inst}_r[\mathcal{D}_k \vdots \Delta_k] \models M@t$. Hence, by Expression (4), there are a substitution σ and intervals $\varrho_1, \ldots, \varrho_n$ such that $(M_1\sigma@\varrho_1, \ldots, M_n\sigma@\varrho_n) \in \mathsf{inst}_r[\mathcal{D}_k \vdots \Delta_k]$ and $M'\sigma@(\varrho_1 \cap \cdots \cap \varrho_n) \models M@t$. Since $(M_1\sigma@\varrho_1, \ldots, M_n\sigma@\varrho_n)$ belongs to $\mathsf{inst}_r[\mathcal{D}_k \vdots \Delta_k]$, the sequence $(M_1\sigma@\varrho_1, \ldots, M_n\sigma@\varrho_n)$ belongs also to $\mathsf{inst}_r[\mathcal{D}_k]$. Therefore, by Expression (2), we obtain that $\mathcal{D}_k \models M_i@\varrho_i$, for each $i \in \{1, \ldots, n\}$. Hence, by definition, $T_\Pi(\mathfrak{I}_{\mathcal{D}_k}) \models M'\sigma@(\varrho_1 \cap \cdots \cap \varrho_n)$, and so, $T_\Pi(\mathfrak{I}_{\mathcal{D}_k}) \models M@t$. By the inductive assumption, we have $\mathfrak{I}_{\mathcal{D}_k} \subseteq T_\Pi^k(\mathfrak{I}_\mathcal{D})$, therefore $T_\Pi^{k+1}(\mathfrak{I}_\mathcal{D}) \models M@t$. ◻

Theorem 2 (Completeness): Consider Procedure 2 running on input Π and $\mathcal{D}$. For each $k \in \mathbb{N}$, upon the completion of the kth iteration of the loop of Procedure 2, it holds that $T_\Pi^k(\mathfrak{I}_\mathcal{D}) \subseteq \mathfrak{I}_{\mathcal{D}'}$.

Proof. We use the same notation $\mathcal{N}_k$, Δ_k, and $\mathcal{D}_k$ as in the proof of Theorem 1. Let α be the least ordinal such that $T_\Pi^\alpha(\mathfrak{I}_\mathcal{D}) = \mathfrak{C}_{\Pi,\mathcal{D}}$, so $T_\Pi^\alpha(\mathfrak{I}_\mathcal{D}) = T_\Pi^{\alpha+1}(\mathfrak{I}_\mathcal{D})$. We will show, inductively on natural numbers $k \leqslant \alpha$, that $T_\Pi^k(\mathfrak{I}_\mathcal{D}) \subseteq \mathfrak{I}_{\mathcal{D}_k}$. The base case holds trivially, because we have $\mathcal{D}_0 = \mathcal{D}$, and so, $T_\Pi^0(\mathfrak{I}_\mathcal{D}) \subseteq \mathfrak{I}_{\mathcal{D}_0}$.

For the inductive step, assume that $T_\Pi^{k+1}(\mathfrak{I}_\mathcal{D}) \models M@t$, for some relational fact $M@t$. We show that $\mathfrak{I}_{\mathcal{D}_{k+1}} \models M@t$. If $T_\Pi^k(\mathfrak{I}_\mathcal{D}) \models M@t$ then, by the inductive assumption, $\mathfrak{I}_{\mathcal{D}_k} \models M@t$. Since $\mathfrak{I}_{\mathcal{D}_k} \subseteq \mathfrak{I}_{\mathcal{D}_{k+1}}$, we obtain that $\mathfrak{I}_{\mathcal{D}_{k+1}} \models M@t$. Now, assume that $T_\Pi^k(\mathfrak{I}_\mathcal{D}) \not\models M@t$ and $T_\Pi^{k+1}(\mathfrak{I}_\mathcal{D}) \models M@t$. Hence, there exists a rule $r \in \Pi$, say of the form $M' \leftarrow M_1 \wedge \cdots \wedge M_n$, and a time point t' such that an application of r at t' yields $M@t$. More precisely, it means that there exists a substitution σ such that $T_\Pi^k(\mathfrak{I}_\mathcal{D}) \models M_i\sigma@t'$, for each $i \in \{1, \ldots, n\}$, and $M'\sigma@t' \models M@t$. Next, for each $i \in \{1, \ldots, n\}$, we let ϱ_i be the subset-maximal interval such that $T_\Pi^k(\mathfrak{I}_\mathcal{D}) \models M_i\sigma@\varrho_i$ and $t' \in \varrho_i$. By the inductive assumption, we obtain that $T_\Pi^k(\mathfrak{I}_\mathcal{D}) \subseteq \mathfrak{I}_{\mathcal{D}_k}$, so $\mathcal{D}_k \models M_i\sigma@\varrho_i$, for each $i \in \{1, \ldots, n\}$. Hence, by Expression (2), we have $(M_1\sigma@\varrho_1, \cdots, M_n\sigma@\varrho_n) \in \mathsf{inst}_r[\mathcal{D}_k]$.

We argue that $(M_1\sigma@\varrho_1, \cdots, M_n\sigma@\varrho_n) \in \mathsf{inst}_r[\mathcal{D}_k \vdots \Delta_k]$. By Expression (4), it suffices to show that there is $i \in \{1, \ldots, n\}$ such that $\mathcal{D}_k \backslash \Delta_k \not\models M_i\sigma@\varrho_i$. For this, we will consider two cases, namely, when $k = 0$ and when $k > 0$.

Case 1: $k = 0$. Then, by the initialisation of Procedure 2, we have $\mathcal{D}_k \backslash \Delta_k = \emptyset$, so $\mathcal{D}_k \backslash \Delta_k \not\models M_i\sigma@\varrho_i$, for each $i \in \{1, \ldots, n\}$.

Case 2: $k > 0$. By Line 7, we have $\mathcal{D}_k = \mathcal{D}_{k-1} \Cup \mathcal{N}_k$ so, by Proposition 1, we obtain $\mathcal{D}_k = \mathcal{D}_{k-1} \Cup \Delta_k$. Thus $\mathfrak{I}_{\mathcal{D}_k} = \mathfrak{I}_{\mathcal{D}_{k-1}} \cup \mathfrak{I}_{\Delta_k}$, so $\mathfrak{I}_{\mathcal{D}_k} \backslash \mathfrak{I}_{\Delta_k} = \mathfrak{I}_{\mathcal{D}_{k-1}} \backslash \mathfrak{I}_{\Delta_k}$, and therefore $\mathfrak{I}_{\mathcal{D}_k} \backslash \mathfrak{I}_{\Delta_k} \subseteq \mathfrak{I}_{\mathcal{D}_{k-1}}$. Hence, it suffices to show that $\mathcal{D}_{k-1} \not\models M_i\sigma@\varrho_i$, for some $i \in \{1, \ldots, n\}$. Suppose towards a contradiction that $\mathcal{D}_{k-1} \models M_i\sigma@\varrho_i$, for all $i \in \{1, \ldots, n\}$. By Theorem 1, $T_\Pi^{k-1}(\mathfrak{I}_\mathcal{D}) \models M_i\sigma@\varrho_i$, for all $i \in \{1, \ldots, n\}$. Thus, $T_\Pi^k(\mathfrak{I}_\mathcal{D}) \models M@t$, which raises a contradiction.

Finally, assume that $k > \alpha$. By the inductive argument above and by Theorem 1, we obtain that $T_\Pi^\alpha(\mathfrak{I}_\mathcal{D}) = \mathfrak{I}_{\mathcal{D}_\alpha}$. Since $T_\Pi^\alpha(\mathfrak{I}_\mathcal{D}) = \mathfrak{C}_{\Pi,\mathcal{D}}$, we obtain that $\mathcal{D}_{\alpha+1} = \mathcal{D}_\alpha$. Thus, by Line 5, we obtain that $\Delta_{\alpha+1} = \emptyset$. Consequently, Procedure 2 terminates in Line 6 in the $\alpha+1$st iteration of the loop, and outputs $\mathcal{D}_\alpha$. Since $T^\alpha_\Pi(\mathfrak{I}_\mathcal{D}) = \mathfrak{I}_{\mathcal{D}_\alpha}$, $T^\alpha_\Pi(\mathfrak{I}_\mathcal{D}) = \mathfrak{C}_{\Pi,\mathcal{D}}$, and $k > \alpha$, we obtain that $T^k_\Pi(\mathfrak{I}_\mathcal{D}) = \mathfrak{I}_{\mathcal{D}_\alpha}$. Hence, $T^k_\Pi(\mathfrak{I}_\mathcal{D}) \subseteq \mathfrak{I}_{\mathcal{D}'}$, where $\mathcal{D}' = \mathcal{D}_\alpha$ is the output of the procedure. □

A.2 Proofs for Section 5

Lemma 1: Consider Procedure 3 running on input Π and $\mathcal{D}$ and let Π_{nr} be the non-recursive fragment of Π. If $flag = 1$, then $\mathfrak{C}_{\Pi_{nr},\mathcal{D}} \subseteq \mathfrak{I}_{\mathcal{D}'}$.

Proof. If $\mathfrak{C}_{\Pi_{nr},\mathcal{D}}$ entails a relational fact with a recursive predicate, then this fact is already entailed by $\mathfrak{I}_\mathcal{D}$ and, by the form of Procedure 3, we have $\mathfrak{I}_\mathcal{D} \subseteq \mathfrak{I}_{\mathcal{D}'}$. To show that the implication holds also for facts with non-recursive predicates, we observe that $\mathfrak{I}_\mathcal{D} \subseteq \mathfrak{I}_{\mathcal{D}'}$ and $\Pi_{nr} \subseteq \Pi$ imply $\mathfrak{C}_{\Pi_{nr},\mathcal{D}} \subseteq \mathfrak{C}_{\Pi,\mathcal{D}'}$. Hence, it suffices to show that each relational fact with non-recursive predicates which is satisfied in $\mathfrak{C}_{\Pi,\mathcal{D}'}$ is also satisfied in $\mathfrak{I}_{\mathcal{D}'}$.

According to Line 4 and Line 16 in Procedure 3, we know that $\mathcal{D}'$ will change to a bigger dataset $(\mathcal{D}' \Cup \mathcal{N})$ after each iteration, where $\mathcal{N}$ is obtained in Line 3. It suffices to show the case in which $\mathcal{D}'$ and $\mathcal{N}$ denotes the contents of $\mathcal{D}'$ and $\mathcal{N}$ when $flag$ becomes 1 for the first time. Hence, we have $\mathcal{D}'$ and $\mathcal{D}' \Cup \mathcal{N}$ entail same facts with non-recursive predicates in Π according to Line 7. Before the flag changes to 1, Procedure 3 works exactly as Procedure 2, so we obtain that $\mathcal{D}' \Cup \mathcal{N} = T_\Pi(\mathfrak{I}_{\mathcal{D}'})$. Therefore, it suffices to show that both $T_\Pi(\mathfrak{I}_{\mathcal{D}'})$ and $T^2_\Pi(\mathfrak{I}_{\mathcal{D}'})$ entails the same facts with non-recursive predicates in Π, as applying the same argument recursively implies $\mathfrak{I}_{\mathcal{D}'}$ and $\mathfrak{C}_{\Pi,\mathcal{D}'}$ entail same facts with non-recursive predicates in Π. Towards a contradiction we suppose that $T^2_\Pi(\mathfrak{I}_{\mathcal{D}'}) \models M@t$ for some relational fact $M@t$ with a non-recursive predicate in Π and $T_\Pi(\mathfrak{I}_{\mathcal{D}'}) \not\models M@t$. Hence, there is a rule $r \in \mathsf{ground}(\Pi,\mathcal{D})$ and a time point t' such that $T^1_\Pi(\mathfrak{I}_{\mathcal{D}'})$ entails each body atom of r at t', and the head of r holding at t' entails $M@t$. Since $M@t$ is a relational fact with a non-recursive predicate in Π and according to Definition 5, we obtain that there is no path with a cycle ending in the non-recursive predicate node representing M, so we obtain that each body atom in r should mentions only non-recursive predicates in Π. Recall that $\mathfrak{I}_{\mathcal{D}'}$ and $T^1_\Pi(\mathfrak{I}_{\mathcal{D}'})$ entails same facts with non-recursive predicates in Π, so $\mathfrak{I}_{\mathcal{D}'}$ also entails each body atom of r at t', so $T^1_\Pi(\mathfrak{I}_{\mathcal{D}'}) \models M@t$. Hence, $\mathfrak{I}_{\mathcal{D}'} \models M@t$, which raises a contradiction.

Lemma 2: If $\mathfrak{I}_\mathcal{D} \mid_{(-\infty,t]} = T_\Pi(\mathfrak{I}_\mathcal{D}) \mid_{(-\infty,t]}$, for a forward propagating program Π, dataset $\mathcal{D}$, and time point t, then $\mathfrak{I}_\mathcal{D} \mid_{(-\infty,t]} = \mathfrak{C}_{\Pi,\mathcal{D}} \mid_{(-\infty,t]}$.

Proof. It suffices to show that $T_\Pi(\mathfrak{I}_\mathcal{D}) \mid_{(-\infty,t]} = T^2_\Pi(\mathfrak{I}_\mathcal{D}) \mid_{(-\infty,t]}$, as applying the same argument recursively implies $\mathfrak{I}_\mathcal{D} \mid_{(-\infty,t]} = \mathfrak{C}_{\Pi,\mathcal{D}} \mid_{(-\infty,t]}$. The inclusion $T_\Pi(\mathfrak{I}_\mathcal{D}) \mid_{(-\infty,t]} \subseteq T^2_\Pi(\mathfrak{I}_\mathcal{D}) \mid_{(-\infty,t]}$ is clear, so we proceed with the opposite direction. Towards a contradiction we suppose that $T^2_\Pi(\mathfrak{I}_\mathcal{D}) \models M@t'$ and $T_\Pi(\mathfrak{I}_\mathcal{D}) \not\models M@t'$, for some relational fact $M@t'$ with $t' \in (-\infty, t]$. Hence, there is a rule $r \in \mathsf{ground}(\Pi,\mathcal{D})$ and a time point t'' such that $T_\Pi(\mathfrak{I}_\mathcal{D})$ satisfies each body atom of r at t'', and the head of r holding at t'' entails $M@t'$. Since r is forward-propagating, we have $t'' \leqslant t'$ so, by $t' \leqslant t$, we obtain that $t'' \leqslant t$. Moreover, by the fact that r mentions only past operators in its body, we obtain that already $T_{\Pi}(\mathfrak{I}_{\mathcal{D}}) \mid_{(-\infty,t]}$ satisfies each body atom of r at t''. However, by the assumption, $\mathfrak{I}_{\mathcal{D}} \mid_{(-\infty,t]} = T_{\Pi}(\mathfrak{I}_{\mathcal{D}}) \mid_{(-\infty,t]}$, so $\mathfrak{I}_{\mathcal{D}} \mid_{(-\infty,t]}$ satisfies each body atom of r at t'', and so, $T_{\Pi}(\mathfrak{I}_{\mathcal{D}}) \models M@t'$, which raises a contradiction.

Lemma 3: If in Procedure 3 a rule r is removed from Π' in Line 10 or in Line 15, then $\mathfrak{C}_{\Pi',\mathcal{D}'\Cup\mathcal{N}} = \mathfrak{C}_{\Pi'\setminus\{r\},\mathcal{D}'\Cup\mathcal{N}}$.

Proof. Clearly, $\mathfrak{C}_{\Pi',\mathcal{D}'\Cup\mathcal{N}} \supseteq \mathfrak{C}_{\Pi'\setminus\{r\},\mathcal{D}'\Cup\mathcal{N}}$, therefore it is sufficient to show that $\mathfrak{C}_{\Pi',\mathcal{D}'\Cup\mathcal{N}} \subseteq \mathfrak{C}_{\Pi'\setminus\{r\},\mathcal{D}'\Cup\mathcal{N}}$. Suppose towards a contradiction that $\mathfrak{C}_{\Pi',\mathcal{D}'\Cup\mathcal{N}} \nsubseteq \mathfrak{C}_{\Pi'\setminus\{r\},\mathcal{D}'\Cup\mathcal{N}}$, so there is the least ordinal α such that $T^{\alpha+1}_{\Pi'}(\mathfrak{I}_{\mathcal{D}'\Cup\mathcal{N}}) \models M@t$ and $T^{\alpha+1}_{\Pi'\setminus\{r\}}(\mathfrak{I}_{\mathcal{D}'\Cup\mathcal{N}}) \not\models M@t$, for some relational fact $M@t$. So, there is a substitution σ and a time point t' such that—for r of the generic form $M' \leftarrow M_1 \wedge \cdots \wedge M_n$—we have $T^{\alpha}_{\Pi'}(\mathfrak{I}_{\mathcal{D}'\Cup\mathcal{N}}) \models M_i\sigma@t'$, for all $i \in \{1,\ldots,n\}$, and $M'@t' \models M@t$.

If in Line 10, the condition in the **if** statement applies, then r has a body atom M_i that is non-recursive in Π and such that $\mathcal{D}' \not\models M_i\sigma@t'$. Since $\mathfrak{C}_{\Pi_{nr},\mathcal{D}} \subseteq \mathfrak{I}_{\mathcal{D}'}$, we get $\mathfrak{C}_{\Pi_{nr},\mathcal{D}} \not\models M_i\sigma@t'$, and so, $\mathfrak{C}_{\Pi,\mathcal{D}} \not\models M_i\sigma@t'$. Moreover, as $\Pi' \subseteq \Pi$ and $\mathcal{D}' \Cup \mathcal{N} = T_{\Pi'}(\mathfrak{I}_{\mathcal{D}'})$, we obtain $T^{\alpha}_{\Pi'}(\mathfrak{I}_{\mathcal{D}'\Cup\mathcal{N}}) \subseteq \mathfrak{C}_{\Pi,\mathcal{D}}$. Thus $T^{\alpha}_{\Pi'}(\mathfrak{I}_{\mathcal{D}'\Cup\mathcal{N}}) \not\models M_i\sigma@t'$, which raises a contradiction.

Now, if in Line 15, the condition in the **if** statement applies, then Π' is forward-propagating. By the construction of t_r and the fact that $r \in \Pi'$ is forward-propagating, we obtain that $t' \leqslant t_r$. As all $M_i\sigma@t'$ hold in $T^{\alpha}_{\Pi'}(\mathfrak{I}_{\mathcal{D}'\Cup\mathcal{N}})$, all these facts hold also in $\mathfrak{I}_{\mathcal{D}'}$. Therefore, $\mathfrak{I}_{\mathcal{D}'\Cup\mathcal{N}} \models M@t$ so $T^{\alpha+1}_{\Pi'\setminus\{r\}}(\mathfrak{I}_{\mathcal{D}'\Cup\mathcal{N}}) \models M@t$, which raises a contradiction.

Theorem 3 (Soundness and Completeness): Consider Procedure 3 running on input Π and $\mathcal{D}$. For each $k \in \mathbb{N}$, the partial materialisation $\mathcal{D}'$ obtained upon completion of the kth iteration of the main loop represents the interpretation $T^{k}_{\Pi}(\mathfrak{I}_{\mathcal{D}})$.

Proof. If for both Line 10 and Line 15, the condition in the IF statement does not apply, then Procedure 3 works in the same way as Procedure 2 so, by Theorems 1 and 2, $\mathcal{D}'$ represents $T^{k}_{\Pi}(\mathfrak{I}_{\mathcal{D}})$. If $flag$ is changed to 1 in the kth iteration, then $\mathfrak{C}_{\Pi_{nr},\mathcal{D}} \subseteq \mathfrak{I}_{\mathcal{D}'}$, by Lemma 1. Therefore, $\mathfrak{C}_{\Pi_{nr},\mathcal{D}} = \mathfrak{C}_{\Pi\setminus\Pi_{nr},\mathcal{D}'}$, and so, Π_{nr} can be safely deleted from Π in Line 8.

Otherwise, the loop from Procedure 3 works similarly as Procedure 2, except that it deletes in Line 10 or Line 15 rules. As we have shown in Lemma 3, such rules can be safely deleted from the program, without loosing the properties established in Theorems 1 and 2.

A.3 Program from Experiments in Section 6

ResearchAssistantCandidate(x) ← $\boxminus_{[0,5]}$ UndergraduateStudent(x)

ResearchAssistantCandidate(x) ← $\diamondminus_{[0,2]}$GraduateStudent(x)

ResearchAssistantCandidate(x) ← $\boxplus_{[0,2]}$TeachingAssistant(x)

ResearchAssistant(x) ← undergraduateDegreeFrom(x,y) ∧ $\boxminus_{[0,3]}$ResearchAssistantCandidate(x)

ResearchAssistant(x) ← mastersDegreeFrom(x,y) ∧ $\boxminus_{[0,1]}$ResearchAssistantCandidate(x)

LecturerCandidate(x) ← $\boxminus_{[0,2]}$ResearchAssistant(x)

LecturerCandidate(x) ← $\boxminus_{[0,4]}$ResearchAssistantCandidate(x)

LecturerCandidate(x) ← $\boxminus_{[0,1]}$GraduateStudent(x) ∧ publicationAuthor(y,x)$\mathcal{S}_{(0,1]}$Publication(y)

Lecturer(x) ← LecturerCandidate(x)$\mathcal{U}_{(0,2]}$researchInterest(x,y)

Lecturer(x) ← $\boxplus_{[1,5]}$LecturerCandidate(x)

AssistantProfessorCandidate(x) ← $\diamondminus_{[1,3]}$Lecturer(x)

AssistantProfessorCandidate(x) ← $\boxminus_{[1,2]}$LecturerCandidate(x) ∧ $\diamondplus_{[0,3]}$publicationAuthor(z,x)

AssistantProfessorCandidate(x) ← $\boxminus_{[1,2]}$LecturerCandidate(x) ∧ $\diamondminus_{[0,3]}$doctoralDegreeFrom(x,y)

AssociateProfessorCandidate(x) ← $\boxminus_{[1,3]}$Lectuer(x) ∧ $\diamondminus_{[0,3]}$doctoralDegreeFrom(x,y) ∧ publicationAuthor(y,x)

AssociateProfessorCandidate(x) ← $\boxminus_{[1,5]}$AssistantProfessorCandidate(x)

AssociateProfessorCandidate(x) ← $\boxminus_{[1,3]}$AssistantProfessor(x)

AssociateProfessorCandidate(x) ← $\boxminus_{[1,2]}$AssistantProfessorCandidate(x) ∧ doctoralDegreeFrom(x,y)

AssociateProfessor(x) ← $\diamondminus_{[1,2]}$AssociateProfessorCandidate(x))

AssociateProfessorCandidate(x) ← $\boxplus_{[1,3]}$AssistantProfessor(x)

FullProfessorCandidate(x) ← $\boxplus_{[1,2]}$AssociateProfessorCandidate(x) ∧ $\diamondminus_{[0,3]}$publicationAuthor(y,x)

FullProfessorCandidate(x) ← $\boxminus_{[1,2]}$AssociateProfessor(x) ∧ $\diamondminus_{[0,3]}$publicationAuthor(y,x)

GoodDepartment(y) ← $\boxminus_{[0,2]}$worksFor(x,y) ∧ FullProfessor(x)

SmartStudent(x) ← UndergraduateStudent(x) ∧ $\diamondplus_{[1,2]}$memberOf(x,y) ∧ GoodDepartment(y)

SmartStudent(x) ← GraduateStudent(x) ∧ $\diamondplus_{[1,3]}$memberOf(x,y) ∧ GoodDepartment(y)

GoodDepartment(x) ← $\boxplus_{[0,2]}$SmartStudent(x) ∧ $\diamondminus_{[0,1]}$publicationAuthor(y,x)

ScientistCandidate(x) ← worksFor(x,y) ∧ $\diamondminus_{[0,1]}$ScientistCandidate(x)

Scientist(x) ← $\boxminus[0,4]$ScientistCandidate(x)

Scientist(x) ← $\diamondminus_{[1,2]}$FullProfessor(x)

FullProfessor(x) ← $\diamondminus_{[1,2]}$Scientist(x)

University(x1)← mastersDegreeFrom(x,x1)

hasAlumnus(x,y)← degreeFrom(y,x)

Faculty(x)← Professor(x)

Person(x1)← member(x,x1)

Professor(x)← Chair(x)

Person(x)← degreeFrom(x,x1)

Person(x1)← hasAlumnus(x,x1)

member(x,y)← memberOf(y,x)

University(x)← hasAlumnus(x,x1)

Organization(x1)← subOrganizationOf(x,x1)

Person(x)← Employee(x)

Organization(x)← member(x,x1)

Faculty(x)← Lecturer(x)

Professor(x1)← advisor(x,x1)

Professor(x)← FullProfessor(x)

TeachingAssistant(x)← teachingAssistantOf(x,x1)

Organization(x)← University(x)

University(x1)← doctoralDegreeFrom(x,x1)

University(x1)← undergraduateDegreeFrom(x,x1)

Person(x)← GraduateStudent(x)

Student(x)← UndergraduateStudent(x)

Publication(x)← publicationAuthor(x,x1)

Organization(x)← ResearchGroup(x)

Person(x)← Chair(x)

Person(x)← TeachingAssistant(x)

Person(x)← emailAddress(x,x1)

Employee(x)←a1:Person(x) ∧ worksFor(x,x1) ∧ Organization(x1)

TeachingAssistant(x)← Person(x) ∧ teachingAssistantOf(x,x1) ∧ Course(x1)

Organization(x)← subOrganizationOf(x,y) ∧ Person(x) ∧ Student(x)

degreeFrom(x,y)← hasAlumnus(y,x)

Employee(x)← Faculty(x)

Professor(x)← AssociateProfessor(x)

Professor(x)← AssistantProfessor(x)

worksFor(x,y)← headOf(x,y)

University(x1)← degreeFrom(x,x1)

memberOf(x,y)← member(y,x)

Course(x1)← teacherOf(x,x1)

Person(x)← telephone(x,x1)

memberOf(x,y)← worksFor(x,y)

Person(x)← advisor(x,x1)

Organization(x)← Department(x)

Person(x1)← publicationAuthor(x,x1)

Work(x)← Course(x)

degreeFrom(x,y)← doctoralDegreeFrom(x,y)

Person(x)← undergraduateDegreeFrom(x,x1)

Person(x)← doctoralDegreeFrom(x,x1)

Course(x1)← teachingAssistantOf(x,x1)

degreeFrom(x,y)← mastersDegreeFrom(x,y)

Person(x)← ResearchAssistant(x)

degreeFrom(x,y)← undergraduateDegreeFrom(x,y)

Person(x)← mastersDegreeFrom(x,x1)

Faculty(x)← teacherOf(x,x1)

Course(x)← GraduateCourse(x)

Person(x)← Student(x)

Chair(x)← Person(x) ∧ headOf(x,x1) ∧ Department(x1)

Student(x)← Person(x) ∧ takesCourse(x,x1) ∧ Course(x1)