Problem 1. *[4]*
Dimension: $n = 2$,
Component functions:
$f_1(\boldsymbol{x}) = \max\{f_1^1(\boldsymbol{x}), f_1^2(\boldsymbol{x}), f_1^3(\boldsymbol{x})\} + f_2^1(\boldsymbol{x}) + f_2^2(\boldsymbol{x}) + f_2^3(\boldsymbol{x})$,
$f_2(\boldsymbol{x}) = \max\{f_2^1(\boldsymbol{x}) + f_2^2(\boldsymbol{x}), f_2^2(\boldsymbol{x}) + f_2^3(\boldsymbol{x}), f_2^1(\boldsymbol{x}) + f_2^3(\boldsymbol{x})\}$,
$f_1^1(\boldsymbol{x}) = x_1^4 + x_2^2,\ f_1^2(\boldsymbol{x}) = (2 - x_1)^2 + (2 - x_2)^2, f_1^3(\boldsymbol{x}) = 2e^{-x_1+x_2}$,
$f_2^1(\boldsymbol{x}) = x_1^2 - 2x_1 + x_2^2 - 4x_2 + 4,\ f_2^2(\boldsymbol{x}) = 2x_1^2 - 5x_1 + x_2^2 - 2x_2 + 4$,
$f_2^3(\boldsymbol{x}) = x_1^2 + 2x_2^2 - 4x_2 + 1$,
Starting point: $\boldsymbol{x}_0 = (2, 2)^T$,
Optimum point: $\boldsymbol{x}^* = (1, 1)^T$,
Optimum value: $f^* = 2$.

Problem 2. *[4]*
Dimension: $n = 2$,
Component functions: $f_1(\boldsymbol{x}) = |x_1 - 1| + 200 \max\{0, |x_1| - x_2\}$,
$f_2(\boldsymbol{x}) = 100(|x_1| - x_2)$,
Starting point: $\boldsymbol{x}_0 = (-1.2, 1)^T$,
Optimum point: $\boldsymbol{x}^* = (1, 1)^T$,
Optimum value: $f^* = 0$.

Problem 3. *[4]*
Dimension: $n = 4$,
Component functions: $f_1(\boldsymbol{x}) = |x_1 - 1| + 200 \max\{0, |x_1| - x_2\}$
$+180 \max\{0, |x_3| - x_4\} + |x_3 - 1| + 10.1(|x_2 - 1| + |x_4 - 1|) + 4.95|x_2 + x_4 - 2|$,
$f_2(\boldsymbol{x}) = 100(|x_1| - x_2) + 90(|x_3| - x_4) + 4.95|x_2 - x_4|$,
Starting point: $\boldsymbol{x}_0 = (1, 3, 3, 1)^T$,
Optimum point: $\boldsymbol{x}^* = (1, 1, 1, 1)^T$,
Optimum value: $f^* = 0$.

Problem 4. *[4]*
Dimension: n = *2, 5, 10, 50, 100, 150, 200, 250, 350, 500, 750,*
Component functions: $f_1(\boldsymbol{x}) = n \max\{|x_i| :\ i = 1, \ldots, n\}$, $\quad f_2(\boldsymbol{x}) = \sum_{i=1}^n |x_i|$,
Starting point: $\boldsymbol{x}_0 = (i,\ i = 1, \ldots, \lfloor n/2 \rfloor,\ \ -i,\ i = \lfloor n/2 \rfloor + 1, \ldots, n)^T$,
Optimum point: $\boldsymbol{x}^* = (x_1^*, \ldots, x_n^*)^T$, $x_i^* = \alpha$ *or* $x_i^* = -\alpha$, $\alpha \in \mathbb{R}$, $i = 1, \ldots, n$,
Optimum value: $f^* = 0$.

Problem 5. *[4]*
Dimension: n = *2, 5, 10, 50, 100, 150, 200, 250, 300, 350, 400, 500, 1000, 1500, 3000, 10 000, 15 000, 20 000, 50 000,*
Component functions: $f_1(\boldsymbol{x}) = 20 \max\left\{\left|\sum_{i=1}^n (x_i - x_i^*) t_j^{i-1}\right| : j = 1, \ldots, 20\right\}$,
$f_2(\boldsymbol{x}) = \sum_{j=1}^{20} \left|\sum_{i=1}^n (x_i - x_i^*) t_j^{i-1}\right|$, $t_j = 0.05j$, $j = 1, \ldots, 20$,
Starting point: $\boldsymbol{x}_0 = (0, \ldots, 0)^T$,
Optimum point: $\boldsymbol{x}^* = (1/n, \ldots, 1/n)^T$,
Optimum value: $f^* = 0$.

Problem 6.
Dimension: $n = 2$
Component functions: $f_1(\boldsymbol{x}) = x_2 + 0.1(x_1^2 + x_2^2) + 10 \max\{0, -x_2\}$,
$f_2(\boldsymbol{x}) = |x_1| + |x_2|$,
Starting point: $\boldsymbol{x}_0 = (10, 1)^T$,
Optimum point: $\boldsymbol{x}^* = (5, 0)^T$,
Optimum value: $f^* = -2.5$.

Problem 7.
Dimension: $n = 2$
Component functions: $f_1(\boldsymbol{x}) = |x_1 - 1| + 200 \max\{0, |x_1| - x_2\}$
$+ 10 \max\{x_1^2 + x_2^2 + |x_2|, x_1 + x_1^2 + x_2^2 + |x_2| - 0.5, |x_1 - x_2| + |x_2| - 1, x_1 + x_1^2 + x_2^2\}$,
$f_2(\boldsymbol{x}) = 100(|x_1| - x_2) + 10(x_1^2 + x_2^2 + |x_2|)$,
Starting point: $\boldsymbol{x}_0 = (-2, 1)^T$,
Optimum point: $\boldsymbol{x}^* = (0.5, 0.5)^T$,
Optimum value: $f^* = 0.5$.

Problem 8.
Dimension: $n = 3$
Component functions: $f_1(\boldsymbol{x}) = 9 - 8x_1 - 6x_2 - 4x_3 + 2|x_1| + 2|x_2| + 2|x_3|$
$+ 4x_1^2 + 2x_2^2 + 2x_3^2 + 10 \max\{0, x_1 + x_2 + 2x_3 - 3, -x_1, -x_2, -x_3\}$,
$f_2(\boldsymbol{x}) = |x_1 - x_2| + |x_1 - x_3|$,
Starting point: $\boldsymbol{x}_0 = (0.5, 0.5, 0.5)^T$,
Optimum point: $\boldsymbol{x}^* = (0.75, 1.25, 0.25)^T$,
Optimum value: $f^* = 3.5$.

Problem 9.
Dimension: $n = 4$
Component functions: $f_1(\boldsymbol{x}) = x_1^2 + (x_1 - 1)^2 + 2(x_1 - 2)^2 + (x_1 - 3)^2 + 2x_2^2$
$+ (x_2 - 1)^2 + 2(x_2 - 2)^2 + x_3^2 + (x_3 - 1)^2 + 2(x_3 - 2)^2 + (x_3 - 3)^2$
$+ 2x_4^2 + (x_4 - 1)^2 + 2(x_4 - 2)^2$
$f_2(\boldsymbol{x}) = \max\{(x_1 - 2)^2 + x_2^2, (x_3 - 2)^2 + x_4^2\}$
$+ \max\{(x_1 - 2)^2 + (x_2 - 1)^2, (x_3 - 2)^2 + (x_4 - 1)^2\}$
$+ \max\{(x_1 - 3)^2 + x_2^2, (x_3 - 3)^2 + x_4^2\} + \max\{x_1^2 + (x_2 - 2)^2, x_3^2 + (x_4 - 2)^2\}$
$+ \max\{(x_1 - 1)^2 + (x_2 - 2)^2, (x_3 - 1)^2 + (x_4 - 2)^2\}$,
Starting point: $\boldsymbol{x}_0 = (4, 2, 4, 2)^T$,
Optimum point: $\boldsymbol{x}^* = (7/3, 1/3, 0.5, 2)$,
Optimum value: $f^* = 11/6$.

Problem 10.
Dimension: $n = 2, 4, 5, 10, 20, 50, 100, 150, 200$
Component functions: $f_1(\boldsymbol{x}) = \sum_{i=1}^n x_i^2, \quad f_2(\boldsymbol{x}) = \sum_{i=2}^n |x_i - x_{i-1}|$,
Starting point: $\boldsymbol{x}_0 = (x_{0,1}, \dots, x_{0,n})^T$, $x_{0,i} = 0.1i$,
Optimum point: For even n: $\boldsymbol{x}^* = (x_1^*, \dots, x_n^*)^T$*:* $x_1^* = -0.5$, $x_n^* = 0.5$,
$x_{2i}^* = 1$, $i = 1, \dots, n/2 - 1$, $x_{2i+1}^* = 1$, $i = 1, \dots, n/2 - 1$,
For odd $n \geq 3$*:* $x^* = (x_1^*, \dots, x_n^*)^T$*:* $x_1^* = -0.5$, $x_n^* = 0.5$, $x_j^* = 0$,
$j = \lfloor n/2 \rfloor + 1$, $x_{2i}^* = 1$, $x_{2i+1}^* = -1$, *for* $2i \leq \lfloor n/2 \rfloor$, $x_{2i}^* = -1$,
$x_{2i+1}^* = 1$, *for* $2i > \lfloor n/2 \rfloor + 1$,
Optimum value: For even n: $f^* = 1.5 - n$,
For odd $n \geq 3$*:* $f^* = 2.5 - n$.

probList_coax

1. *Dem–Mal*: $f(x) = \max\{5x_1 + x_2, -5x_1 + x_2, x_1^2 + x_2^2 + 4x_2\}$; $x^{(0)} = (1,1)$; $x^* = (0,-3)$; $f^* = -3$.
2. *Mifflin*: $f(x) = -x_1 + 20\max\{x_1^2 + x_2^2 - 1, 0\}$; $x^{(0)} = (0.8, 0.6)$; $x^* = (1,0)$; $f^* = -1$.
3. *LQ*: $f(x) = \max\{-x_1 - x_2, -x_1 - x_2 + x_1^2 + x_2^2 - 1\}$; $x^{(0)} = (-0.5, -0.5)$; $x^* = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$; $f^* = -\sqrt{2}$.
4. *MAXQ*: $f(x) = \max\limits_{1\le i\le 20}\{x_i^2\}$; $x_i^{(0)} = 0,\ i = 1,\dots,10$; $x_i^{(0)} = -i,\ i = 11,\dots,20$; $x^* = (0,\dots,0)$; $f^* = 0$.
5. *QL*: $f(x) = \max\limits_{1\le i\le 3} f_i(x)$; $f_1(x) = x_1^2 + x_2^2$; $f_2(x) = x_1^2 + x_2^2 + 10(-4x_1 - x_2 + 4)$, $f_2(x) = x_1^2 + x_2^2 + 10(-x_1 - 2x_2 + 6)\}$; $x^{(0)} = (-1, 5)$; $x^* = (1.2, 2.4)$; $f^* = 7.2$.
6. *CB2*: $f(x) = \max\{x_1^2 + x_2^4, (2 - x_1)^2 + (2 - x_2)^2, 2e^{(-x_1 + x_2)}\}$; $x^{(0)} = (1, -0.1)$; $x^* = (1.1392286, 0.899365)$; $f^* = 1.9522245$.
7. *CB3*: $f(x) = \max\{x_1^4 + x_2^2, (2 - x_1)^2 + (2 - x_2)^2, 2e^{(-x_1 + x_2)}\}$; $x^{(0)} = (2,2)$; $x^* = (1,1)$; $f^* = 2$.

[redacted] An Academic Test Problem ACAD

The essential aim of examining the subsequent academic test problem is to investigate how often the algorithms under consideration converge to the known minimum of the objective function and not just to a critical point. This test setting is inspired by Example 3.1 in [4].

For the objective function $f : \mathbb{R}^2 \to \mathbb{R}$ with

$$f(x, y) := x^2 + y^2 + x + y - |x| - |y| \qquad x, y \in \mathbb{R},$$

the DC-composition $f := g - h$ to be examined is chosen as

$$g(x, y) := \frac{3}{2}(x^2 + y^2) + x + y, \qquad h(x, y) := |x| + |y| + \frac{1}{2}(x^2 + y^2) \qquad x, y \in \mathbb{R},$$

so that the component functions g, h are uniformly convex. The global minimum is at $(-1, -1)$, but there exist three additional critical and non-optimal points $(-1, 0)$, $(0, -1)$, and $(0, 0)$. To investigate the ability of the different solvers to find the optimal point, 10,000 test runs for