True expression of the FIM (i is the index over examples, s is the index over spatial positions)

$$\begin{aligned}
&\frac{1}{n}\sum_i\left(\sum_{s_1}x_{is_1}\otimes g_{is_1}\right)\left(\sum_{s_2}x_{is_2}^\top\otimes g_{is_2}^\top\right)\\
=&\frac{1}{n}\sum_i\sum_{s_1}\sum_{s_2}\left(x_{is_1}\otimes g_{is_1}\right)\left(x_{is_2}^\top\otimes g_{is_2}^\top\right)\\
=&\frac{1}{n}\sum_i\sum_{s_1}\sum_{s_2}x_{is_1}x_{is_2}^\top\otimes g_{is_1}g_{is_2}^\top
\end{aligned}$$

We study the following expression:

$$(*) =\frac{1}{n}\sum_i\sum_{s_1}\sum_{s_2}\left(x_{is_1}x_{is_2}^\top - E\left[xx^\top\right]\right)\otimes\left(g_{is_1}g_{is_2}^\top - E\left[gg^\top\right]\right)$$

Here E denotes the averaged value over the discrete sum:

$$E\left[xx^\top\right] = \frac{1}{n}\frac{1}{|S|}\sum_i\sum_s x_{is}x_{is}^\top$$

$$\begin{aligned}
(*) =\ & \underbrace{\left[\frac{1}{n}\sum_i\sum_{s_1}\sum_{s_2}x_{is_1}x_{is_2}^\top\otimes g_{is_1}g_{is_2}^\top\right]}_{(1)} - \underbrace{\left[\frac{1}{n}\sum_i\sum_{s_1}\sum_{s_2}x_{is_1}x_{is_2}^\top\otimes\frac{1}{n}\frac{1}{|S|}\sum_i\sum_s g_{is}g_{is}^\top\right]}_{(2)}\\
&-\underbrace{\left[\frac{1}{n}\frac{1}{|S|}\sum_i\sum_s x_{is}x_{is}^\top\otimes\frac{1}{n}\sum_i\sum_{s_1}\sum_{s_2}g_{is_1}g_{is_2}^\top\right]}_{(3)} + \underbrace{\left[\frac{1}{n}\sum_j\sum_{s_1}\sum_{s_2}\left(\frac{1}{n}\frac{1}{|S|}\sum_i\sum_s x_{is}x_{is}^\top\right)\otimes\left(\frac{1}{n}\frac{1}{|S|}\sum_i\sum_s g_{is}g_{is}^\top\right)\right]}_{(4)}
\end{aligned}$$

$$(4) = \frac{1}{|S|}\left(\frac{1}{n}\sum_i\sum_s x_{is}x_{is}^\top\right)\otimes\left(\frac{1}{n}\sum_i\sum_s g_{is}g_{is}^\top\right)$$

Spatially uncorrelated features SUA (cf KFC paper):

$$\sum_{s_1}\sum_{s_2}g_{is_1}g_{is_2}^\top = \sum_s g_{is}g_{is}^\top$$

Thus (3) = (4) and they cancel out.

Assuming $(*) = 0$, we obtain:

$$\begin{aligned}
(1) &= (2)\\
\frac{1}{n}\sum_i\sum_{s_1}\sum_{s_2}x_{is_1}x_{is_2}^\top\otimes g_{is_1}g_{is_2}^\top &= \frac{1}{n}\sum_i\sum_{s_1}\sum_{s_2}x_{is_1}x_{is_2}^\top\otimes\frac{1}{n}\frac{1}{|S|}\sum_i\sum_s g_{is}g_{is}^\top
\end{aligned}$$

If we additionnally require that (case A):

$$\sum_{s_1}\sum_{s_2}x_{is_1}x_{is_2}^\top = \sum_s x_{is}x_{is}^\top$$

then we obtain KFC:

$$\boxed{\frac{1}{n}\sum_i\sum_{s_1}\sum_{s_2}x_{is_1}x_{is_2}^\top\otimes g_{is_1}g_{is_2}^\top = \frac{1}{n}\sum_i\sum_s x_{is}x_{is}^\top\otimes\frac{1}{n}\frac{1}{|S|}\sum_i\sum_s g_{is}g_{is}^\top}$$

But we could alternatively assume that (case B):

$$\sum_{s_1}\sum_{s_2} x_{is_1}x_{is_2}^\top \quad = \quad |S|\sum_{s} x_{is}x_{is}^\top$$

or anything in-between.

In case B, the expression differs by a multiplicative factor $|S|$:

$$\boxed{\frac{1}{n}\sum_i\sum_{s_1}\sum_{s_2} x_{is_1}x_{is_2}^\top \otimes g_{is_1}g_{is_2}^\top \quad = \quad \frac{1}{n}\sum_i\sum_s x_{is}x_{is}^\top \otimes \frac{1}{n}\sum_i\sum_s g_{is}g_{is}^\top}$$