COORDINATE FRAMES OF THE U.S. SPACE OBJECT CATALOGS

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Abstract

Using analytical theories known as General Perturbations (GP), the US Space Command maintains databases of artificial satellite orbital elements known as the Space Object Catalog. The GP Catalog may be more familiar to the general public as NORAD or NASA two-line orbital elements. These elements are referenced to an unconventional reference frame based on an approximation to the uniform equinox of date. This frame is rarely discussed in detail because the uncertainty of the GP orbital elements is usually larger than the subtle distinctions between non-standard Catalog frames and the conventional International Astronomical Union (IAU) frames. Advances in computing technology now allow for Space Catalog maintenance using higher accuracy numerical techniques, sometimes called Special Perturbations (SP). The SP Catalog is also maintained with respect to a temporal equinox. However, pending IAU resolutions will formally redefine conventional Earth orientation theory with new models that do not require an equinox. The authors review the fundamentals of the old and new IAU transformations, including the transformation of satellite velocity, and relate the current Space Catalog frames to these conventions. Because the uniform equinox is already uncommon outside the US Space Commands, and because it will be rendered obsolete by upcoming IAU conventions, reliance on the uniform equinox should be phased out for high accuracy applications such as the SP Space Catalog.

The US Space Catalog

The United States Department of Defense (DoD) has maintained a database of satellite states since the launch of the first Sputnik in 1957, known as the Space Object Catalog, or simply the Space Catalog.[1] These satellite states are regularly updated with observations from the Space Surveillance Network, a globally distributed network of interferometer, radar and optical tracking systems.[2] Two separate catalog databases are maintained under the US Space Command: a primary catalog by the Air Force Space Command (AFSPC), and an alternate catalog by the Naval Space Command (NSC). The number of cataloged objects is approaching 10,000.

Different astrodynamic theories are used to maintain these catalogs. The so-called General Perturbations (GP) theory provides a general analytical solution of the satellite equations of motion.[3] The orbital elements and their associated partial derivatives are expressed as series expansions in terms of the initial conditions of the differential equations. The GP theories operated efficiently on the earliest electronic computing machines, and were therefore adopted as the primary theory for Space Catalog orbit determination. Assumptions must be made to simplify these analytical theories, such as truncation of the Earth's gravitational potential to a few zonal harmonic terms.[4] The atmosphere is often modeled as a static density field that exponentially decays.[5] Third body influences and resonance effects are partially modeled.[6]

NASA maintains civilian databases of GP orbital elements, also known as the NASA or NORAD two-line elements.[7] These GP elements are "mean" elements that have specific periodic features removed to enhance long-term prediction performance. They require special software to reconstruct the compressed trajectory.[8,9]

The GP reference frame has been a recurring source of confusion for analysts and satellite owner-operators because this frame has not been precisely defined in a form accessible to a larger audience. This astrodynamic reference frame is not widely recognized outside the US Space Commands, and the adopted military descriptors are also unconventional. Because the GP elements are not terribly accurate, misinterpretation of this reference frame is probably within the uncertainty of the orbital elements themselves and has not warranted distinction.[10]

General Perturbations theory is not sufficiently accurate to support every mission, and increasing the accuracy of analytical GP theory usually requires significant development efforts.[11] Higher accuracy methods, sometimes known as Special Perturbations (SP), follow from the direct integration of the satellite equations of motions using numerical methods.[12] Although SP orbit determination is more computationally intensive than GP theory, it is simpler to develop and more readily modified to afford improvements in accuracy.

An SP catalog is now feasible because computer capacity has grown much faster than the size of the Space Catalog.[13] In 1997, the Naval Research Laboratory and the NSC demonstrated this concept by maintaining the entire Space Catalog for one month using a distributed machine architecture.[14] This processor, known as SPeCIAL-K, passed Initial Operational Capability in July 1999 and is expected to be certified as having Final Operational Capability in mid-2000. In September 1999, the AFSPC began using an SP orbit determination processor known as the Astrodynamics Support Workstation (ASW) to maintain its "high accuracy catalogue".[15,16] These supplemental SP catalogs will eventually supersede the operational GP catalogs.

The SP Space Catalogs are a relatively new product. The standards regarding the general distribution and format of SP satellite states are evolving as anticipated military and civilian applications stipulate new requirements. SP Catalog states already support certain civilian missions such as collision avoidance for the Space Shuttle and the International Space Station.[17] The increased accuracy will likely expand the potential customer base for SP satellite states.

At the time of this writing, the Space Catalogs are maintained and distributed with respect to more than one coordinate frame. Confusion of these frames could significantly degrade the perceived accuracy of the SP Catalog. The best use of the GP and SP catalogs requires a full understanding of these frames and the terminology surrounding them.

Coordinate Systems And Frames

The ordinary differential equations that describe satellite motion are most simply defined in a Newtonian-inertial space. In this case, the time derivatives of the coordinate axes are negligible, eliminating Coriolis effects. However, satellite motion is usually observed from stations fixed to the surface of the rotating Earth. Thus for geocentric orbit determination, the Earth's orientation with respect to inertial space must be well approximated.

The motion of the Earth's spin axis with respect to celestial objects has been refined through astrometric observation over many decades. The temporal equinox has long been the basis of these observations and their corresponding celestial reference systems. It is therefore convenient to adopt a celestial reference system as the Newtonian-inertial space for orbit determination. By so doing, the satellite state is tied to a specific celestial convention and a specific realization of its temporal equinox and pole.

A conventional coordinate *system* is a set of prescriptions that defines a triad of orthogonal axes at any time, whereas a conventional *frame* is the practical realization of a system at a specific epoch based on the system prescriptions.[18] Conventions need not be exact; rather, they are practical representations that are commonly recognized. Conventional reference systems are approved by the International Astronomical Union (IAU) and partly maintained by the International Earth Rotation Service (IERS). Before 1998, the Fundamental Katalog 5 (FK5) was the basis of the IAU celestial reference system.[19] The FK5 theory is defined by the IAU 1976 Precession model and IAU 1980 Theory of Nutation, and was primarily realized from observations taken at optical wavelengths.

Beginning January 1, 1998, the IAU adopted the International Celestial Reference System (ICRS).[20] The ICRS defines a triad of orthogonal axes whose origin resides at the barycenter of the solar system. Its axes are realized by observations of extragalactic radio sources from the Very Long Baseline Interferometry (VLBI) network, and the origin in right ascension is based on the FK5 J2000 value adopted for radio source 3C 273.[21,22] At optical wavelengths, the ICRS has been astrometrically tied to the HIPPARCOS star catalog to maintain continuity with the FK5 system.[23] As estimates of the relative positions between the defining sources improves, and as more defining sources are added, the frame of the ICRS (known as the ICRF) will be maintained such that there is no net rotation introduced with respect to previous realizations. A significant difference between the ICRF and the FK5 theory is that the directions of the FK5 axes move as a function of time with respect to inertial space, while the directions of the ICRF axes remain fixed for all practical purposes.

The geocentric counterpart to the ICRF will be known as the Geocentric Celestial Reference Frame (GCRF). It has been the celestial reference frame for the IERS since January 1, 1997.[24] The axes of the GCRF are close to the frame of FK5 J2000 to provide continuity between the former and current IAU systems. Because there was no official IAU nutation theory compatible with the ICRS when it was adopted, the IERS has continued to maintain tabulated corrections for the IAU FK5 theory to relate it to the GCRF.[25] Applying these corrections effectively defines a different (more accurate) conventional theory than the FK5 convention. At the time of this writing, the IERS corrections at J2000* are $\delta\Delta\psi \approx$ -0.05077 arcseconds and $\delta\Delta\varepsilon \approx$ -0.00237 arcseconds.[26] The relationship between the GCRF and the FK5 J2000 frame is then approximated by the quasi-orthogonal rotation:

$$\mathbf{r}_{\mathrm{GCRF}} = \begin{bmatrix} 1 & 0.000274\times10^{-8} & -9.790527\times10^{-8} \\ -0.000274\times10^{-8} & 1 & -1.147780\times10^{-8} \\ 9.790527\times10^{-8} & 1.147780\times10^{-8} & 1 \end{bmatrix} \mathbf{r}_{\mathrm{FK5}} \tag{1}$$

where

$\mathbf{r}_{\mathrm{GCRF}}$ is a three dimensional vector with respect to the GCRF basis, and

$\mathbf{r}_{\mathrm{FK5}}$ is a three dimensional vector with respect to the FK5 basis at date J2000.

The change of basis implied by Eq. (1) is within the formal uncertainty of the FK5 prescription (50 mas or 1.5 m/ER† in pole, and 80 mas or 2.5 m/ER in equinox).[27] Neither frame is exactly coincident with the best estimates of the pole and equinox at J2000, as these directions are determined by observation, rather than convention.[28]

Orbit determination requires both celestial references frames (which define the Newtonian-inertial space in which the differential equations of satellite motion are valid), and terrestrial references frames (from which the satellite observations are taken). The conventional terrestrial frame, known as the International Terrestrial Reference Frame (ITRF), has its origin at the center of mass of the Earth.‡ Its axes are realized by the adopted coordinates of defining fiducial stations on the surface of the Earth. The relative station coordinates are affected by plate tectonic motion on the order of centimeters per year, such that they are annually re-estimated through VLBI, Satellite Laser Ranging (SLR), GPS, and DORIS observations.[29] The ITRF is a weighted, global combination of several analysis center solutions, adjusted such that there is no net rotation or frame shift with respect to previous realizations of the ITRF.

The WGS-84 terrestrial frame is primarily used by the US DoD.[30] It is realized through GPS observations, although the fundamental WGS-84 stations are usually constrained by their adopted ITRF coordinates during solution. Thus, the modern WGS-84 and ITRF terrestrial frames agree at the few cm level. Within the uncertainty of the WGS-84 frame, they are practically identical.

The terrestrial frame is related to the celestial frame through a series of rotations known as an Earth orientation model. The complete model is sometimes divided into partial sequences of rotations, where intermediate frames are defined between these partial sequences. There are two conventional Earth orientation models: the classical transformation and the non-rotating origin transformation. The latter is expected to be the basis of the upcoming IAU theory, but the Space Catalog frames are currently defined as intermediate bases within the classical transformation.

Classical Transformation

The classical transformation between a celestial basis and a terrestrial or "Earth fixed" basis is well established. This transformation, also known as Option 1, takes the following vector-matrix form:[31]

$$\mathbf{r}(t_{\mathrm{i}})_{\mathrm{TRF}} = [\mathbf{W}(t_{\mathrm{i}})]\,[\mathbf{R}(t_{\mathrm{i}})]\,[\mathbf{N}(t_{\mathrm{i}})]\,[\mathbf{P}(t_{\mathrm{i}},t_0)]\,\mathbf{r}(t_0)_{\mathrm{CRF}} \tag{2}$$

where

$\mathbf{r}(t_{\mathrm{i}})_{\mathrm{TRF}}$ is a three dimensional position vector with respect to the terrestrial frame of date t_{i},

$\mathbf{r}(t_0)_{\mathrm{CRF}}$ is a three dimensional position vector with respect to the celestial frame of date t_0,

$[\mathbf{P}(t_{\mathrm{i}},t_0)] = \mathbf{ROT3}(-z_A)\,\mathbf{ROT2}(\theta_A)\,\mathbf{ROT3}(-\zeta_A)$ is the precession matrix from date t_0 to t_{i},

$[\mathbf{N}(t_{\mathrm{i}})] = \mathbf{ROT1}(-\varepsilon_A-\Delta\varepsilon)\,\mathbf{ROT3}(-\Delta\psi)\,\mathbf{ROT1}(\varepsilon_A)$ is the nutation matrix of date t_{i},

$[\mathbf{R}(t_{\mathrm{i}})] = \mathbf{ROT3}(\theta_{\mathrm{GST}})$ is the sidereal rotation matrix of date t_{i}, and

$[\mathbf{W}(t_{\mathrm{i}})] = \mathbf{ROT2}(-x_{\mathrm{p}})\,\mathbf{ROT1}(-y_{\mathrm{p}})$ is the polar motion matrix of date t_{i}.

θ_{GST} is a function of Universal Time UT1 (the primary measure of Earth rotation), and **ROT1**, **ROT2**, and **ROT3** are rotations about the X, Y, and Z axes respectively.[32,33] The time dependent angles ζ_A, θ_A, z_A, ε_A, $\Delta\psi$, $\Delta\varepsilon$, and θ_{GST} are defined in the IERS Conventions 1996 compatible with FK5 theory. Tabulated values of x_{p}, y_{p}, and UT1–UTC are known as Earth Orientation Parameters (EOP), and are available through the IERS.[34,35] Because the matrices of Eq. (2) are orthogonal, the inverse transformation is simply

$$\mathbf{r}(t_0)_{\mathrm{CRF}} = [\mathbf{P}(t_{\mathrm{i}},t_0)]^{\mathrm{T}}\,[\mathbf{N}(t_{\mathrm{i}})]^{\mathrm{T}}\,[\mathbf{R}(t_{\mathrm{i}})]^{\mathrm{T}}\,[\mathbf{W}(t_{\mathrm{i}})]^{\mathrm{T}}\,\mathbf{r}(t_{\mathrm{i}})_{\mathrm{TRF}}\,.$$

The classical transformation separates the motion of precession from nutation to predict the nominal direction of the Earth's rotational pole with respect to the "fixed stars" on the celestial sphere. This

* That is, the epoch of J2000 – January 1, 2000 12^{h} TT.

† Small angles are usually supplemented here in units of distance per Earth radii, such as cm/ER. "mas" implies milliarcseconds.

‡ In future definitions of the ITRF datum, it will be necessary to modify the basic transformation to account for cm level geocenter motion with respect to the fiducial stations (see IERS Gazette No. 50).

rotational pole is currently known as the Celestial Ephemeris Pole (CEP).[36] Precession **[P]** describes the large-scale secular motion of the pole while nutation **[N]** describes the quasi-periodic variability of the CEP with respect to its precessional drift. A simple sidereal rotation **[R]** from the temporal equinox about the CEP largely fulfills the change of basis from a celestial to a terrestrial framework. An additional set of small-angle rotations known as polar motion **[W]** compensates for the fact that the CEP moves with respect to the ITRF in a slow and somewhat unpredictable way.[37]

In concept, the equinox is defined by the line of intersection between the plane of the equator and the plane of the ecliptic. The equator is the plane perpendicular to the CEP passing through the center of mass of the central body—Earth. The equator can be either the "mean equator" (the plane defined by the pole before **[N]** is applied) or the "true equator" (the plane defined by the pole after **[N]** is applied). The equinox is therefore described as a "mean" or "true" equinox of date, depending upon the equator and pole to which is it referenced.

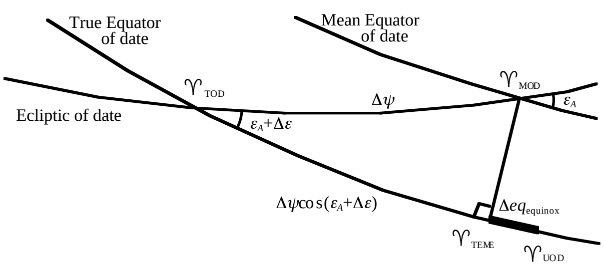

Figure 1. The Mean, True, and Uniform Equinoxes of Date, as viewed from outside the Celestial Sphere (not to scale).

The "uniform equinox" is defined by the true equinox of date minus the so-called "Equation of the Equinoxes" ($EQ_{equinox}$).[38] The uniform equinox is not widely recognized as a basis for satellite states because it is not an equinox in the sense defined above. To realize the uniform equinox basis within the classical transformation, one must sub-divide the Greenwich Sidereal angle θ_{GST} into Greenwich Mean Sidereal time θ_{GMST} and $EQ_{equinox}$. Then Eq. (2) becomes

$$\mathbf{r}(t_i)_{TRF} = [\mathbf{W}(t_i)]\,[\Re(t_i)]\,[\mathbf{Q}(t_i)]\,[\mathbf{N}(t_i)]\,[\mathbf{P}(t_i,t_0)]\,\mathbf{r}(t_0)_{CRF} \quad (3)$$

where

$$[\Re] = \mathbf{ROT3}(\theta_{GMST}) \text{ and } [\mathbf{Q}] = \mathbf{ROT3}(EQ_{equinox})\ .$$

Historically, the uniform equinox was seen as the projection of the mean equinox of date onto the true equator of date. For this reason, it is sometimes cited in early astrodynamics literature as the "true equator and mean equinox of date" (TEME)*, if it is specified at all.[39] It is usually described as a rotation with respect to the mean-of-date equinox (Figure 1). For example, AOES defines this TEME rotation as[40]

$$\mathbf{r}_{TEME} = \begin{bmatrix} 1 & 0 & -\Delta\psi\sin(\varepsilon_A+\Delta\varepsilon) \\ 0 & 1 & -\Delta\varepsilon \\ \Delta\psi\sin(\varepsilon_A+\Delta\varepsilon) & \Delta\varepsilon & 1 \end{bmatrix} \mathbf{r}_{MOD}$$

which is seen as a small angle approximation to the series of rotations

$$\mathbf{r}_{TEME} = \mathbf{ROT1}(-90°)\,\mathbf{ROT3}(-\Delta\psi\sin(\varepsilon_A+\Delta\varepsilon))\,\mathbf{ROT1}(90°-\Delta\varepsilon)\,\mathbf{r}_{MOD}\ . \quad (4)$$

At the milliarcsecond level, Eq.(4) is numerically equivalent to the mean-of-date to uniform-of-date transformation implied by Eq. (3):

$$\mathbf{r}_{UOD} = \mathbf{ROT3}(EQ_{equinox})\,\mathbf{ROT1}(-\varepsilon_A-\Delta\varepsilon)\,\mathbf{ROT3}(-\Delta\psi)\,\mathbf{ROT1}(\varepsilon_A)\,\mathbf{r}_{MOD}\ . \quad (5)$$

However, a distinction now exists between the geometric and kinematic interpretations of the Equation of the Equinoxes.[41] Effective January 1, 1997:

$$EQ_{equinox} = \theta_{GST} - \theta_{GMST} = \Delta\psi\cos(\varepsilon_A+\Delta\varepsilon) + \Delta eq_{equinox}\ ,$$
$$\Delta eq_{equinox} = 2.64\text{mas}\cdot\sin(\Omega) - 0.009\text{mas}\cdot\sin(2\Omega)$$

where Ω is the mean longitude of the ascending node of the Moon. Under this kinematic interpretation, the rotation from the mean-of-date equinox basis to the uniform-equinox basis has an approximate small angle form:

$$\mathbf{r}_{UOD} = \begin{bmatrix} 1 & \Delta eq_{equinox} & -\Delta\psi\sin(\varepsilon_A+\Delta\varepsilon) \\ -\Delta eq_{equinox} & 1 & -\Delta\varepsilon \\ \Delta\psi\sin(\varepsilon_A+\Delta\varepsilon) & \Delta\varepsilon & 1 \end{bmatrix} \mathbf{r}_{MOD}$$

The kinematic correction to the Equation of the Equinoxes $\Delta eq_{equinox}$ is numerically significant within these small angle matrices. The long term difference between kinematic and geometric interpretations is about an order of magnitude larger than the higher order terms neglected in the small angle forms.

For the sake of clarity, the term "uniform equinox" is hereafter reserved for the intermediate basis between **[ℜ]** and **[Q]** in Eq. (3), consistent with the kinematic interpretation. The term "true equator and mean equinox of date" (TEME) is considered to describe the former geometric interpretation without the $\Delta eq_{equinox}$ correction. Both frames share the CEP as the Z axis. The intermediate bases resulting from the classical

* This term is a misnomer. "True equator" correctly implies the CEP as the Z axis, but the X axis is not the conventional "mean equinox".

TABLE 1
FRAME DESCRIPTORS OF THE CLASSICAL TRANSFORMATION OF DATE t_i

Abbrv.	General Designators	Other Designators*	Eq. (3) Rotations
TEF	(True) Earth fixed, body fixed	Earth Centered Rotating (ECR)	⇦ $[\mathbf{W}(t_i)]$
PEF	Pseudo Earth fixed, Pseudo body fixed	Earth Fixed Greenwich (EFG) Earth Centered Earth Fixed (ECEF)	⬀ ⇦ $[\Re(t_i)]$
UOD	Uniform (Equinox) of Date	Earth Centered Inertial (ECI) True Equator and Mean Equinox	⬀ ⇦ $[\mathbf{Q}(t_i)]$
TOD	True (Equinox) of Date, True Equator and True Equinox		⬀ ⇦ $[\mathbf{N}(t_i)]$
MOD	Mean (Equinox) of Date, Mean Equator and Mean Equinox		⬀ ⇦ $[\mathbf{P}(t_i,t_{J2000})]$
J2000	Mean (Equinox) of 2000		⬀ ⇦ $[\text{Eq.1}]^T$
GCRF	Geocentric Celestial Reference Frame		⬀

* AFSPC Operating Instruction 60-102 11-Mar-1996, TP SCC 008

transformation are sometimes described by the naming conventions in Table 1.

The conventional transformation of velocity is not explicitly prescribed by the IERS Conventions 1996, but it is partly implied by the time derivatives of Eq. (2) or Eq. (3). Because the UOD, MOD, and TOD intermediate frames are Newtonian-inertial at epoch, $\frac{d}{dt}[\mathbf{Q}]$ $\frac{d}{dt}[\mathbf{N}]$ and $\frac{d}{dt}[\mathbf{P}]$ are zero by definition.

The time derivative of Eq. (2) and Eq. (3) implies the use of $\dot{\theta}_{GMST}$ in $\frac{d}{dt}[\Re]$ and $\dot{\theta}_{GST}$ in $\frac{d}{dt}[\mathbf{R}]$, respectively. As a result, $\dot{\theta}_{GMST}$ is often used to define the Earth's angular velocity. $\dot{\theta}_{GMST}$ is equal to $7.292115855307\times10^{-5}$ rad/sec on January 1, 2000 0^h UT1. However, $\dot{\theta}_{GMST}$ defines the Earth's rotation rate with respect to the precessing mean equinox, not inertial space. Sidereal rotation and rate are directly measured with respect to a quasi-inertial space through the observations of stars, quasi-stellar radio sources, and satellite motion. For this reason, and because all realizations of the equinox are considered inertial by the practiced convention, the angular velocity of the Earth with respect a Newtonian-inertial frame should be used in $\frac{d}{dt}[\Re]$ or $\frac{d}{dt}[\mathbf{R}]$.

The "stellar angle" θ represents the angle of a reference meridian on the Earth with respect to a celestial non-rotating origin (or departure point) in the plane of the movable equator. It is derived from the conventional expression of θ_{GMST} as a function of UT1:[42,43]

$$\theta = 2\pi\,(0.779057273264 + 1.00273781191135448 \cdot T_u \cdot 36525)\ \text{rad}, \quad (6)$$

where T_u is Julian Centuries of UT1. Its time derivative with respect to a uniform time scale is

$$\omega = 7.29211514670698\times10^{-5} \cdot (1 - LOD/86400)\ \text{rad/sec}. \quad (7)$$

LOD represents the instantaneous rate of change of UT1 with respect to a uniform time scale (such as UTC or TAI). It is called excess length of day (in units of seconds) and it is estimated or predicted by the IERS. Like UT1, it is subject to sub-millisecond zonal tide variations. Eq. (7) is consistent with the angular velocity defined in the Explanatory Supplement to IERS Bulletins A and B.[44]

Although $\dot{\theta}_{GMST}$ is less compatible with an inertial equinox, the difference is small relative to the requirements of most Earth fixed satellite applications. However, the difference between $\dot{\theta}_{GMST}$ and ω is about as large as the average effect of *LOD* ($\sim10^{-11}$ rad/sec). Applications that use length of day corrections will likely require ω in lieu of $\dot{\theta}_{GMST}$.

Velocity due to polar motion $\frac{d}{dt}[\mathbf{W}]$ calls for the numerical derivatives of the tabulated angles provided by the IERS. Polar motion rates are nominally smaller than UT1 rate by two orders of magnitude and are not directly provided by the IERS. For these reasons, $\frac{d}{dt}\mathbf{[W]}$ is usually neglected. Based on these conventions, the classical transformation of satellite velocity is:

$$\mathbf{v}(t_i)_{TRF} = \mathbf{[W}(t_i)\mathbf{]}\ [\Re(t_i)]\ \mathbf{[Q}(t_i)\mathbf{]}\ \mathbf{[N}(t_i)\mathbf{]}\ \mathbf{[P}(t_i,t_0)\mathbf{]}\ \mathbf{v}(t_0)_{CRF} + \mathbf{[W}(t_i)\mathbf{]}\ \frac{d}{dt}[\Re(t_i)]\ \mathbf{[Q}(t_i)\mathbf{]}\ \mathbf{[N}(t_i)\mathbf{]}\ \mathbf{[P}(t_i,t_0)\mathbf{]}\ \mathbf{r}(t_0)_{CRF}$$

where

$\mathbf{v}(t_i)_{TRF}$ is a three dimensional velocity vector with respect to the terrestrial frame of date t_i,

$\mathbf{v}(t_0)_{CRF}$ is a three dimensional velocity vector with respect to the celestial frame of date t_0, and

$$\frac{d}{dt}[\Re] = \begin{bmatrix} -\omega\sin(\theta_{GMST}) & \omega\cos(\theta_{GMST}) & 0 \\ -\omega\cos(\theta_{GMST}) & -\omega\sin(\theta_{GMST}) & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (8)$$

An expression for $\frac{d}{dt}\mathbf{[R]}$, compatible with Eq. (2), would be identical to Eq. (8), except θ_{GMST} is replaced with θ_{GST}.

Non-Rotating Origin Transformation

The classical transformation has an intermediate dependence on the ecliptic of date. However, modern Earth orientation relies on VLBI and SLR observations that are insensitive to the ecliptic plane.[45] This deficiency is alleviated by making explicit use of a "non-rotating origin" instead of the temporal equinox.[46] In matrix-vector form, this alternative transformation appears similar to the classical transformation:

$$\mathbf{r}(t_i)_{TRF} = \mathbf{[W'}(t_i)\mathbf{]}\ \mathbf{[R'}(t_i)\mathbf{]}\ \mathbf{[N'P}(t_i)\mathbf{]}\ \mathbf{r}_{CRF} \quad (9)$$

where

$\mathbf{r}(t_i)_{TRF}$ is a three dimensional position vector with respect to a terrestrial frame at date t_i,

$\mathbf{r}_{CRF}$ is a three dimensional position vector with respect to a (time invariant) celestial frame,

$$\mathbf{[N'P}(t_i)\mathbf{]} = \mathbf{ROT3}(-s)\begin{bmatrix} 1-aX^2 & -aXY & -X \\ -aXY & 1-aY^2 & -Y \\ X & Y & 1-a(X^2+Y^2) \end{bmatrix}$$

is the precession-nutation matrix at date t_i,

$\mathbf{[R'}(t_i)\mathbf{]} = \mathbf{ROT3}(\theta)$ is the sidereal rotation matrix using the stellar angle at date t_i, and

$\mathbf{[W'}(t_i)\mathbf{]} = \mathbf{ROT2}(-x_p)\ \mathbf{ROT1}(-y_p)\ \mathbf{ROT3}(s')$ is the polar motion matrix at date t_i.

The time varying quantities a, s, θ, s', X and Y will be defined according to the IERS Conventions 2000 (stellar angle θ was defined in Eq. (6)). The Earth Orientation Parameters x_p, y_p, and UT1–UTC will be continue to be tabulated by the IERS. X and Y are the direction cosines of the celestial pole with respect to the GCRF.

The non-rotating origin transformation, known as Option 2, is kinematically correct as it segregates the terrestrial and celestial motion of the pole from the rotation of the Earth. This method is computationally efficient and conceptually simple relative to the classical transformation. The equinox is notably absent from the non-rotating origin transformation, since the ecliptic plane is purposely omitted.

The transformation of velocity takes the same form as the classical transformation, namely

$$\mathbf{v}(t_i)_{TRF} = \mathbf{[W'}(t_i)\mathbf{]}\ \mathbf{[R'}(t_i)\mathbf{]}\ \mathbf{[N'P}(t_i)\mathbf{]}\ \mathbf{v}_{CRF} + \mathbf{[W'}(t_i)\mathbf{]}\ \frac{d}{dt}\mathbf{[R'}(t_i)\mathbf{]}\ \mathbf{[N'P}(t_i)\mathbf{]}\ \mathbf{r}_{CRF}$$

if the velocity due to polar motion is ignored. The form of $\frac{d}{dt}\mathbf{[R']}$ will be identical to that of $\frac{d}{dt}[\Re]$ of Eq. (8), if θ replaces θ_{GMST}.

Frame of the General Perturbations Catalog

The earliest programmed GP theories were typically referenced to the mean equinox of B1950.0, this fiducial direction being tied to the Fundamental Katalog 4 (FK4) star catalog and its associated system of constants.[47,48] However, the repetitive evaluation of trigonometric functions was a formidable challenge to early digital computers. Out of computational expediency, the early designers of the operational Space Catalog chose to maintain a celestial reference frame close to the TEME of the epoch of the orbital elements.

During GP differential correction, the transformation between the terrestrial frame and the TEME integration frame is only approximate. Both $\Delta\psi$ and $\Delta\varepsilon$ are held constant over the time interval of GP solution to reduce the number of trigonometric evaluations. Specifically, terrestrial observations are related to the inertial integration frame using invariant **[N]** and **[Q]** matrices initially evaluated at the epoch of solution:

$$\mathbf{r}_{TEF}(t_i) = \mathbf{[W}(t_i)\mathbf{]}\ [\Re(t_i)]\ \mathbf{[Q}(t_0)\mathbf{]}\ \mathbf{[N}(t_0)\mathbf{]}\ \mathbf{[P}(t_i,t_0)\mathbf{]}\ \mathbf{[N}(t_0)\mathbf{]^T}\ \mathbf{[Q}(t_0)\mathbf{]^T}\ \mathbf{r}_{TEME}(t_0)\ ,$$

where t_i is the date of the observation and t_0 is the epoch of the integration frame.

The approximation of $\mathbf{[N}(t_i)\mathbf{]}$ and $\mathbf{[Q}(t_i)\mathbf{]}$ with $\mathbf{[Q}(t_0)\mathbf{]}$ and $\mathbf{[N}(t_0)\mathbf{]}$ biases the observational data far from the epoch of solution. The biases remain slight for low Earth satellites having typical fit spans of 3 to 5 days. The effect is more pronounced for geo-synchronous satellites with fit spans approaching one month. Table 2 contains numerical estimates of the uncertainty due to GP frame approximations. Because the IAU 1976 precession model is experiencing secular error growth which changes the statistics over time, this effect was removed from the Table 2 statistics.

TABLE 2
UNCERTAINTIES DUE TO VARIOUS REALIZATIONS OF THE UNIFORM EQUINOX
Assessed w/ Daily LAGEOS States (1988 – 2000)

Frame	Std. Dev. mas (cm/ER)	Max. Error mas (cm/ER)
IERS observed	0.02 (0.05)	-
Small angle matrix	0.2 (0.6)	0.3 (1)
TEME small angle matrix	1.3 (4)	2.7 (8)
106 term FK5* (SP)	3.3 (10)	6.5 (20)
4 term FK5* at t_0 (GP)	45 (140)	110 (350)
4 term FK5* t_i=t_0+1 day	57 (177)	150 (460)
4 term FK5* t_i=t_0+4 days	147 (456)	320 (970)
4 term FK5* t_i=t_0+10 days	216 (669)	490 (1510)
4 term FK5* t_i=t_0+30 days	511 (1580)	960 (2950)

* These FK5 estimates omit the secular trends and biases of the 1976 Precession model (-3.0 mas/yr in $\Delta\psi$, -0.23 mas/yr in $\Delta\varepsilon$), which are statistically significant.

Other GP approximations truncate the FK5 Fourier series to the four largest nutation terms, thus ignoring coefficients whose individual contributions are smaller than 200 mas (6 m/ER).[49] The mean obliquity of the ecliptic is considered constant or truncated to a be linear function of time. Usually, the polar motion matrix **[W]** is ignored or roughly estimated in GP observations.

Frame Of The Special Perturbations Catalog

Through the adoption of modern constants and an unabbreviated FK5 theory, the SP Catalog is more precise in its handling of coordinate frames.[50] The NSC SP software system (SPeCIAL-K) currently conforms to the IAU 1976 Precession and the IAU 1980 Theory of Nutation. It performs numerical integration in the true-of-date frame with the capability to output satellite states in all the intermediate frames listed in Table 1 (except the GCRF). The AFSPC SP software system (ASW) also conforms to the FK5 theory, and is capable of integrating the satellite equations of motion in the J2000 frame. ASW also has options to output states in many frames, including backward compatibility with the approximate GP TEME frame.

At the time of this writing, evidence suggests that the two SP systems maintain different definitions of the Equation of the Equinoxes. SPeCIAL-K's definition of EQ_{equinox} is consistent with the kinematic interpretation of the uniform equinox, while ASW's definition appears consistent with the geometric interpretation of the TEME. The difference is within the uncertainty of the FK5 theory used by both systems, since $\Delta eq_{\text{equinox}}$ never gets larger than 3 mas (10 cm/ER).

Because the Space Catalog is maintained near real time, it requires predicted Earth orientation parameters. The United States Naval Observatory is responsible for time keeping and predicted Earth orientation for the US government and DoD, operating the IERS Sub-bureau for Rapid Service and Predictions. Total Earth orientation is now predicted to an accuracy of about 6.1 mas (19 cm/ER) in standard deviation after four days, with UT1 prediction error dominating the forecast error budget.[51] The FK5 theory is less suitable for real-time high accuracy work, since its errors are now larger than those of short term UT1 forecasts. Accurate EOP forecasting requires improved nutation theories, such as the IERS 1996 Theory of Precession and Nutation, which has an uncertainty of about 0.3 mas (0.9 cm/ER).[52] In contrast, the uncertainty of the IAU FK5 theory is about 3 mas, excluding secular error rates of -3 mas/year (-9 cm/ER/year) in precession and -0.3 mas/year (-0.9 cm/ER/year) in obliquity.[53]

IAU 2000 Earth Orientation Model

IAU Colloquium 180 endorsed the conclusions of the IAU/IUGG Working Group on Non-rigid Earth Nutation Theory, which states that the FK5 theory is not accurate enough for present day needs.[54] It has recommended (in the form of an resolution to be ratified by the IAU XXIV General Assembly in August 2000) that the scientific community replace the IAU 1976 Precession Model and the IAU 1980 Theory of Nutation with the IAU 2000 precession-nutation model as of January 1, 2003.[55] This model will be published in the upcoming IERS Conventions 2000, and will be accurate to 0.2 mas. This model will reference the GCRF.

An alternative IAU 2000B model is planned that will be accurate to the 1 mas level. This model should be no more numerically intensive than the present FK5 theory while providing much higher accuracy. It is expected to rapidly replace the FK5 theory for many satellite applications.

Most importantly, the equinox will no longer be part of the definition of the IAU celestial reference system. Although the final IAU 2000 models have not been published at the time of this writing, it is likely that the recommended transformation between the terrestrial and celestial frames will be based on a non-rotating origin and polar direction cosines with respect to the GCRF. For this reason, there is legitimate concern that the equinox has become an obsolete and inconvenient reference frame for high accuracy orbit determination. Under the new IAU system, the equinox can likely be maintained only through a change of variables or an alternative theory. Alternate methods will be more complex and potentially less accurate than the non-rotating origin formulation.

Implications for the Space Catalog

The Space Catalog supports space object identification, tracking, mission planning and trajectory analysis. The specific frame requirements are usually application dependent. In many cases, the temporal equinox provides no benefit or actually encumbers analyses.

For example, initial acquisition of newly launched Satellite Laser Ranging targets could benefit from an accurate Space Catalog. SLR tracking stations acquire their targets by using narrow beam search patterns around the predicted path of a satellite relative to the observer. Frame errors and low quality predictions affect pointing accuracy. Use of the GP elements impacts the timeliness of initial acquisition, providing fewer SLR observations.

To point a tracking instrument, the satellite state must be transformed to the Earth-fixed frame. Using an intermediate, precessing equinox complicates this process. Specifically, the change of basis from the UOD basis at epoch t_0 to the TEF basis at epoch t_i requires the transformation

$$\mathbf{r}_{TEF}(t_i) = [\mathbf{W}(t_i)]\,[\Re(t_i)]\,[\mathbf{Q}(t_i)]\,[\mathbf{N}(t_i)]\,[\mathbf{P}(t_i,t_0)]\,[\mathbf{N}(t_0)]^{\mathbf{T}}\,[\mathbf{Q}(t_0)]^{\mathbf{T}}\,\mathbf{r}_{UOD}(t_0)\ .$$

However, the transformation from the GCRF to the TEF basis is less involved:

$$\mathbf{r}_{TEF}(t_i) = [\mathbf{W}'(t_i)]\,[\mathbf{R}'(t_i)]\,[\mathbf{N}'\mathbf{P}(t_i)]\,\mathbf{r}_{GCRF}\ .$$

Both the GP and SP Catalogs support rendezvous and collision avoidance. For this critical mission, the "miss distance" of orbiting objects can only be accurately determined from trajectories in the same frame. For a Space Catalog maintained at the uniform equinox of date, the transformation between UOD bases from epoch t_0 to epoch t_i is:

$$\mathbf{r}_{UOD}(t_i) = [\mathbf{Q}(t_i)]\,[\mathbf{N}(t_i)]\,[\mathbf{P}(t_i,t_0)]\,[\mathbf{N}(t_0)]^{\mathbf{T}}\,[\mathbf{Q}(t_0)]^{\mathbf{T}}\,\mathbf{r}_{UOD}(t_0)\ .$$

Of course, this transformation is never required for a Space Catalog maintained with respect to the GCRF.

Space Catalog standards have not required a high level of compliance with international conventions in the past. The imprecise nature of GP theory does not stipulate frame requirements beyond satellite identification purposes. Given the age of the FK5 theory, conformity of the Space Catalog to FK5 constants is relatively recent. Many enhancements have been motivated by the desire to evaluate system performance against alternative sources of high accuracy ephemerides which already accommodate IAU and IERS conventions.[56] Compliance with internationally recognized standards and constants takes greater relevance as more users outside the US Space Commands become interested in the higher accuracy SP Space Catalog.

An adoption of international conventions should simultaneously embrace less ambiguous terminology by the military. For example, the designator "Earth Centered Inertial" (Table 1) usually refers to a generic geocentric-equatorial basis and not the uniform equinox of date.[57] In the past, this "ECI" designator has described coordinate frames based approximate forms of the nutation theory under both the FK4 and FK5 systems. The expression "true equator and mean equinox" is also misleading because the mean equinox is not the X axis of this frame. Arguably, the exact definition of a TEME frame is unclear at the few mas level, due to the kinematic correction to the geometric Equation of the Equinoxes after 1996.

TABLE 3
ERROR DUE TO CONFUSION OF THE EQUINOXES
Assessed w/ Daily LAGEOS States (1988 – 2000)

Frame Difference	Standard Dev. arcsec (m/ER)	Max. Error arcsec (m/ER)
MOD – UOD	7.0 (220)	9.9 (310)
TOD – UOD	8.0 (250)	17.1 (530)

There is concern that the lack of specificity confuses users of NORAD elements into mistaking the TEME frame for either the true-of-date or mean-of-date frame (Table 3). Such confusion can result in a frame bias of several hundred meters per Earth radii, which far exceeds the potential accuracy of a SP catalog.[58]

Modern computers have mitigated the need for the uniform equinox. As the mean, true, and uniform equinoxes are further moved into obsolescence by the IAU, these intermediate frames present an additional layer of operational overhead to high accuracy users conforming with the latest IAU conventions.

Moving Toward IAU Compliance

It is recommended that at least one of the Space Commands maintains SP satellite states with respect to the GCRF to support this upcoming standard. To the level of accuracy of the FK5 theory, the FK5/J2000* frame approximates the GCRF.[59] Switching the default output frame in the SP cataloging software to the J2000 basis will go far to operationally accommodate the GCRF without substantial software modifications to these systems.†

While the FK5/J2000 frame might temporarily serve as a proxy for the GCRF, the IERS currently recommends that the IERS 1996 Theory of Precession and Nutation be used for high accuracy prediction. This theory is based on empirical estimates that absorbed the precession rate errors of the IAU 1976 model.[60] Compatibility with the GCRF requires static corrections to the nutation in longitude and obliquity[61]

$$\Delta\psi_{\mathrm{GCRF}} = \Delta\psi_{\mathrm{IERS1996}} - 43.1\mathrm{mas},$$
$$\Delta\varepsilon_{\mathrm{GCRF}} = \Delta\varepsilon_{\mathrm{IERS1996}} - 5.1\mathrm{mas}$$

Software for the 1996 Theory is available through the IERS, but not all versions are with respect to the GCRF.[62]

Because much of the existing software for the IERS 1996 theory is based upon the classical transformation, it is more complicated than an equivalent non-rotating origin transformation. However, it is possible to apply computation-saving techniques historically employed for classical nutation. The most successful accuracy-preserving technique rearranges the Fourier series in an optimal fashion, and is used today for the FK5 theory.[63,64] Less elaborate techniques truncate the number of terms in the nutation series, or pre-compute precession and nutation and interpolate the theory from tables.[65,66]

Timing trials for these latter methods are listed in Table 4 using linear interpolation and theories available at the time of this writing. The results suggest that interpolation of either the nine **[NP]–[I]** matrix elements (combined precession-nutation minus the identity matrix), or the interpolation of the native nutation variables $\Delta\psi$ and $\Delta\varepsilon$, is faster and potentially more accurate than merely truncating the Fourier series. Final interpolation accuracy will depend on the choice of nodal frequency, significant digits retained in the nodes, and the type and degree of interpolating function.

* The term FK5/J2000 implies the pole and equinox of the FK5 theory (IAU 1976 Precession and 1980 Nutation) at the epoch of J2000.

† While the J2000 frame is accurate to the 50-80 mas level, the GCRF is accurate to the sub-mas level. Use of the term GCRF is best reserved for realizations accurate to the mas level.

TABLE 4
RELATIVE TIME TO EVALUATE
PRECESSION / NUTATION
Assessed w/ 10,000 Randomly Generated States

Theory and Technique	Relative Timing
IERS 1996 (**[NP]–[I]** interpolated)	0.1
IERS 1996 ($\Delta\psi$, $\Delta\varepsilon$ interpolated)	1.0
Full FK5 Theory ($\Delta\psi$, $\Delta\varepsilon$ interpolated)	1.0
Truncated 4 term FK5 Theory (used by GP)	1.2
Full FK5 Theory (used by SP)	3.7
IERS 1996 Theory	11.8

The upcoming IAU 2000A model will certainly require many more floating point operations than the FK5 theory. Because the Space Catalog relies on predicted EOP values that are only accurate to a few mas anyway, the IAU 2000B theory would be a strong candidate to replace the FK5 theory now used by the SP Space Catalog. It would not be difficult to employ the computation-saving techniques in Table 4 for the IAU 2000A theory. The computational expense of interpolation should be nearly the same regardless of theory, once the interpolation tables have been pre-computed.

Summary And Conclusions

The classical Earth orientation theory defines a temporal pole and equinox on the celestial sphere. This method requires clarification of the type of equinox, the associated system of constants, and the frame epoch. The GP Space Catalog is approximately referenced to the uniform equinox and Celestial Ephemeris Pole defined by the FK5 theory at the orbital element epoch. The SP catalog is also based on the FK5 system, but the SP satellite states may be referenced to any one of several intermediate frames within the classical transformation, including the uniform equinox at epoch.

A new Earth orientation model is slated for adoption by the IAU in mid-2000. The IAU 2000A theory will be independent of an equinox and will reference the GCRF. A supplemental IAU 2000B theory will be accurate to the milliarcsecond level and it is expected to evaluate as fast or faster than the existing FK5 theory. The availability of a fast, highly accurate model should accelerate the adoption of the new IAU system and GCRF worldwide.

As SP Catalog availability expands, its temporal equinoxes should be retired and replaced with a reference standard that is widely recognized by both civilian and military agencies and consistent with the upcoming IAU system. The uniform equinox is not widely recognized outside the US Space Commands. The adopted military nomenclature is enigmatic, and maintaining intermediate coordinate systems adds operating costs.

Satellite owner-operators and orbital analysts are anticipating the new IAU theory and the astrodynamics community should expect increasing standardization toward the GCRF. The recent operational SP upgrades to the US Space Catalog provide an excellent opportunity to standardize the SP Catalog to the conventional GCRF as well.

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