5 REFERENCE FRAMES

The following reference frames are used in the context of Earth Observation missions:

Table 3: Earth Observation reference frames usage

Reference frame	Usage Examples
Galactic	Star position and velocities can be given in this reference frame
Barycentric Mean of 1950	Some star catalogues use this reference frame to express the positions of their stars.
Barycentric Mean of 2000	The star catalogues usually use this reference frame to express the positions of their stars.
Heliocentric Mean of 2000	The ephemeris of the planets are usually expressed in this reference frame.
Geocentric Mean of 2000	The FOCC performs the internal calculations related to the predicted and restituted orbits in this reference frame.
Mean of Date	The Mean Local Solar Time is defined in this reference frame.
True of Date	It is the inertial reference frame used for input and output in the CFI software (e.g. star positions).
Earth fixed	It is the reference frame used for input and output of the satellite state vector (i.e. orbit definition), and for the output for geolocation in the CFI software.
Pseudo Earth Fixed	It is similar to Earth Fixed but without considering polar motion rotation.
Topocentric	It is the local horizontal reference frame used to define a looking direction.
Satellite Orbital	It is a reference frame centred in the satellite and defined by the satellite position and velocity. Its used as a reference for the application of the selected attitude control mode.
Satellite Nominal Attitude	It is used for the attitude determination. It is based on relation with the Satellite Orbital frame.
Satellite Attitude	It is used for the attitude determination as well. It is based on relation the Satellite Nominal Attitude frame or on measurements.
Instrument Attitude	It is the reference frame that constitutes the reference for the definition of a look direction relative to the satellite (e.g. to express the pointing of an instrument).

5.1 General Reference Frames

5.1.1 Galactic

The galactic plane is determined by the statistical study of the galactic dynamics. In this reference frame, position are determined by a galactic latitude and longitude. The galactic latitude are taken as the angle measured from the galactic plane, while the galactic longitude are measured from the direction of the galactic centre.

In order to relate the galactic coordinates of a star to its equatorial coordinates, it is necessary to know the position of the galactic pole and the position of the galactic centre. These points have been adopted as follow, for the epoch 1950.0:

Right ascension of the Galactic pole = 12h 49m.

Declination of the Galactic pole = 27º.4.

Galactic longitude of the north celestial pole = 123º (also known as the position angle of the galactic centre)

5.1.2 Barycentric Mean of 1950

It is based on the star catalogue FK4 for the epoch B1950, since the directions of its axes are defined relatively to a given number of that star catalogue positions and proper motions.

The centre of this reference frame is the barycentre of the Solar System. The x-y plane coincides with the predicted mean Earth equatorial plane at the epoch B1950, and the x-axis points towards the predicted mean vernal equinox. The latter is the intersection of the mean equator plane with the mean ecliptic, and the ecliptic is the orbit of the Earth around the Sun. The z-axis points towards north.

The word *mean* indicates that the relatively short periodic nutations of the Earth are smoothed out in the calculation of the mean equator and equinox.

5.1.3 Barycentric Mean of 2000

It is based, according to the recommendations of the International Astronomical Union (IAU), on the star catalogue FK5 for the epoch J2000.0, since the directions of its axes are defined relatively to a given number of that star catalogue positions and proper motions.

The accuracy of this reference system, realized through the FK5 star catalogue, is approximately 0.1’’.

The centre of this reference frame is the barycentre of the Solar System. The x-y plane coincides with the predicted mean Earth equatorial plane at the epoch J2000.0, and the x-axis points towards the predicted mean vernal equinox. The latter is the intersection of the mean equator plane with the mean ecliptic, and the ecliptic is the orbit of the Earth around the Sun. The z-axis points towards north.

The word *mean* indicates that the relatively short periodic nutations of the Earth are smoothed out in the calculation of the mean equator and equinox.

5.1.4 Heliocentric Mean of 2000

It is obtained by a parallel translation of the Barycentric Mean of 2000.0 reference frame from the barycenter of the Solar System to the centre of the Sun.

5.1.5 Geocentric Mean of 2000

It is obtained by a parallel translation of the Barycentric Mean of 2000.0 reference frame from the barycenter of the Solar System to the centre of the Earth.

5.1.6 Mean of Date

The centre of this reference frame is the centre of the Earth. The x-y plane and the x-axis are defined by the mean Earth equatorial plane and the mean vernal equinox of date. The expression *mean of date* means that the system of coordinate axes are rotated with the Earth´s precession from J2000.0 to the date used as epoch. The z-axis points towards north.

The precession of the Earth is the secular effect of the gravitational attraction from the Sun and the Moon on the equatorial bulge of the Earth.

5.1.7 True of Date

The centre of this reference frame is the centre of the Earth. The x-y plane and the x-axis are defined by the true Earth equatorial plane and the true vernal equinox of date. The expression *true of date* indicates the instantaneous Earth equatorial plane and vernal equinox. The transformation from the Mean of Date to the True of Date is the adopted model of the nutation of the Earth.

The nutation is the short periodic effect of the gravitational attraction of the Moon and, to a lesser extent, the planets on the Earth’s equatorial bulge.

5.1.8 Pseudo Earth Fixed

It is defined the same way as Earth Fixed reference frame. The difference is that polar motion rotation is not considered in this case.

5.1.9 Earth Fixed

The Earth fixed reference frame in use is the *IERS Terrestrial Reference Frame* (ITRF).

The zero longitude or IERS Reference Meridian (IRM), as well as the IERS Reference Pole (IRP), are maintained by the International Earth Rotation Service (IERS), based on a large number of observing stations, and define the IERS Terrestrial Reference Frame (ITRF).

5.1.10 Topocentric

Its z-axis coincides with the normal vector to the Earth’s Reference Ellipsoid, positive towards zenith. The x-y plane is the plane orthogonal to the z-axis, and the x-axis and y-axis point positive, respectively, towards east and north.

5.1.11 LIF (Launch Inertial Frame)

The Launch Inertial Frame (LIF) is an Coordinate System whose Z axis and XY plane are the same as Z axis and XY plane of Earth Fixed Coordinate System. It is a user defined Coordinate System as the user needs to provide a longitude and a UTC time to define it. More precisely, X,Y and Z axes are defined as follows:

- X axis: is in the equatorial plane and points where a given user defined meridian was at a given user defined UTC time. The meridian is defined by the user specifying the value of the corresponding longitude (note that, if longitude = 0, the reference is the Greenwich meridian)

- Z axis: is in the direction of the angular velocity of the Earth (= towards North Pole)
- Y axis: complete the right-handed coordinate system

5.2 Satellite Reference Frames

Four levels of reference frames are used for attitude determination:

- The Satellite Orbital frame (SOF)
- Satellite Nominal reference frame (SNRF)
- Satellite reference frame (SRF)
- Instrument reference frame (IRF)

The SOF is used for the computation of the other satellite reference frames (see section 5.2.1 for the definition of this frame)

The SNRF is an ideal attitude model. The axis definition for this frame depends on the attitude model chosen for the satellite. Let’s see two examples:

- Local Normal Pointing attitude (LNP), the z-axis is chosen in the direction of the satellite’s zenith and the x-axis is defined in the direction of the satellite’s inertial velocity vector (in True of Date).
- Yaw Steering Mode attitude (YSM): the z-axis is chosen in the direction of the satellite’s zenith and the x-axis is defined in the direction of the satellite’s velocity vector in the Earth Fixed CS.

A complete list of attitude models can be seen in section 8.1.

The SRF corresponds to the satellite actual (measured) attitude frame. It could be considered as the result of three consecutive rotations of the SNRF over three angles called ***mispointing angles***. The time derivative of those mispointing angles are called ***mispointing rates***.

Finally the IRF is a frame based on an instrument of the satellite. There exists one reference frame per instrument and it is used for location and looking direction from the instrument.

5.2.1 Satellite Orbital

It is a reference frame centred on the satellite and is defined by the Xs, Ys and Zs axes, which are specified relatively to the reference inertial reference frame, namely the True of Date.

The Zs axis points along the radial satellite direction vector, positive from the centre of the TOD reference frame towards the satellite, the Ys axis points along the transversal direction vector within the osculating orbital plane (i.e the plane defined by the position and velocity vectors of the satellite), orthogonal to the Zs axis and opposed to the direction of the orbital motion of the satellite. The Xs axis points towards the out-of-plane direction vector completing the right hand reference frame.

$$\vec{Z}=\frac{\vec{r}}{|\vec{r}|} \qquad \vec{X}=\frac{\vec{r}\wedge\vec{v}}{|\vec{r}\wedge\vec{v}|} \qquad \vec{Y}=\vec{Z}\wedge\vec{X}$$

where $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ are the unitary direction vectors in the (Xs, Ys, Zs) axes, and $\vec{r}$ and $\vec{v}$ are the position and velocity vectors of the satellite expressed in the inertial reference frame.

Next drawing depicts the Satellite Orbital frame:

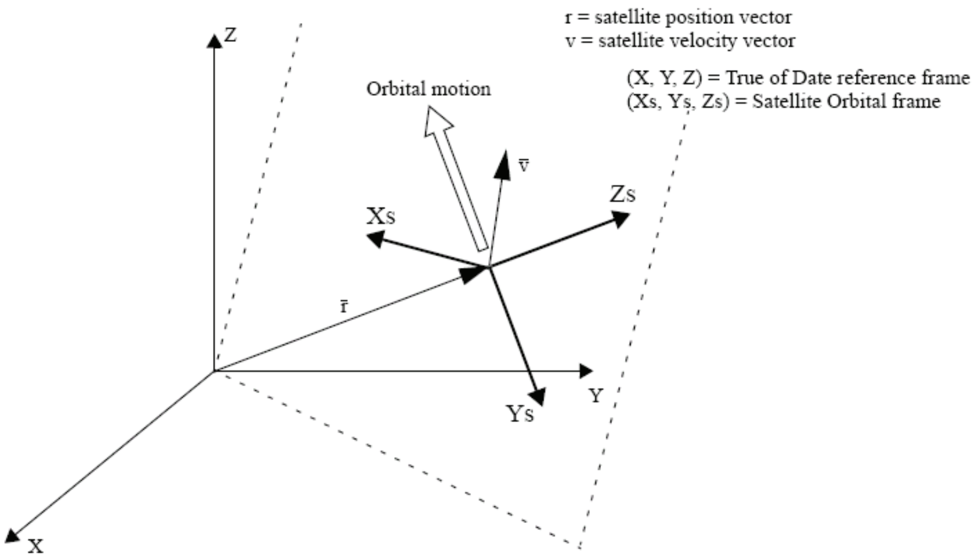
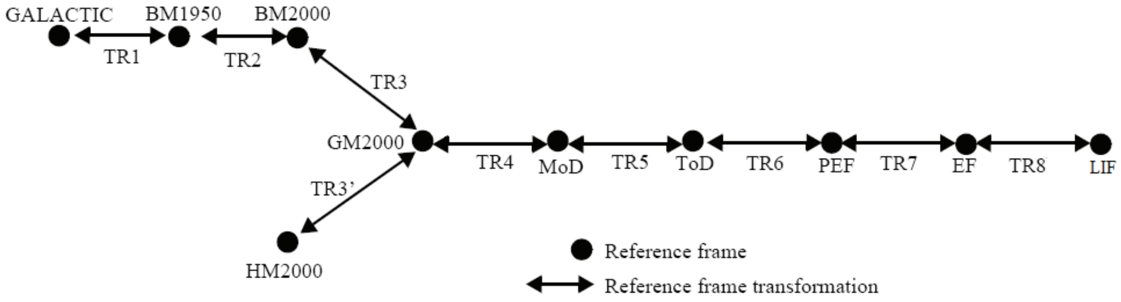


Figure 2: Satellite Orbital Frame

5.3 General Reference Frames Transformations

The following picture identifies the general reference frames transformations that are relevant for the Earth Observation missions.



Reference frames:

GALACTIC = Galactic CS (see section 5.1.1)
BM1950 = Barycentric Mean of 1950.0 (see section 5.1.2)
BM2000 = Barycentric Mean of 2000.0 (see section 5.1.3)
HM2000 = Heliocentric Mean of 2000.0 (see section 5.1.4)
GM2000 = Geocentric Mean of 2000.0 (see section 5.1.5)
MoD = Mean of Date (see section 5.1.6)
ToD = True of Date (see section 5.1.7)
PEF = Pseudo Earth Fixed (see section 5.1.8)
EF = Earth Fixed (see section 5.1.9)
LIF = Launch Inertial Frame (see section 5.1.11)

Transfromations:

TR1 = Galactic to Barycentric Mean of 1950 (see section 5.3.1)
TR2 = Barycentric 1950 to Barycentric 2000 (see section 5.3.2)
TR3 = Solar system barycentre to Earth centre translation (see section 5.3.3)
TR3' = Sun centre to Earth centre translation (see section 5.3.4)
TR4 = Precession (see section 5.3.5)
TR5 = Nutation (see section 5.3.6)
TR6 = Earth rotation + nutation term (see section 5.3.7)
TR7 = Polar motion rotation (see section 5.3.8)
TR8 = Earth rotation

Figure 3: General Reference Frames Transformations

Those transformation are described in the following sections.

Note that whenever a transformation is expressed as a sequence of rotations, the following expressions apply (the angle w is regarded positive):

$$R_X(w)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos w & \sin w\\ 0 & -\sin w & \cos w\end{bmatrix}\qquad R_Y(w)=\begin{bmatrix}\cos w & 0 & -\sin w\\ 0 & 1 & 0\\ \sin w & 0 & \cos w\end{bmatrix}\qquad R_Z(w)=\begin{bmatrix}\cos w & \sin w & 0\\ -\sin w & \cos w & 0\\ 0 & 0 & 1\end{bmatrix}$$

5.3.1 Galactic to Barycentric Mean of 1950

The following picture represents the galactic and the equatorial coordinate systems. The relationship between both systems are given by the equatorial coordinates of the galactic pole for the epoch 1950 and for by the position of the galactic centre.

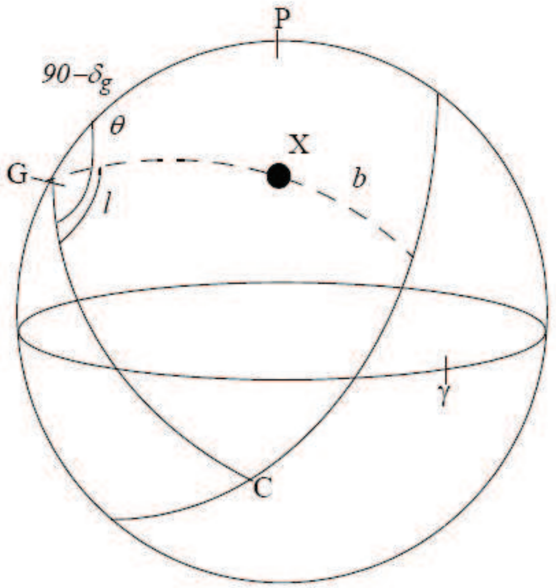


Figure 4: Galactic and Equatorial coordinates

In the figure, considering the spheric triangle GPX, the relationship between the galactic and equatorial coordinates can be established (see GREEN for further details)

$$\cos b \sin(\theta\text{-}l) = \cos\delta \sin(\alpha - \alpha_g)$$

$$\cos b \cos(\theta\text{-}l) = \cos\delta_g \sin\delta - \sin\delta_g \cos\delta \cos(\alpha - \alpha_g)$$

Taking into account the relations between spherical and cartesian coordinates, it is easy to derive the rotation matrix from Galactic to Barycentric B1950.0:

$$R_{(galactic \rightarrow B1950.0)} = \begin{bmatrix} -0{,}06698874 & 0{,}49272847 & -0{,}86760081 \\ -0{,}8727557659 & -0{,}45034696 & -0{,}1883746 \\ -0{,}4835389146 & 0{,}74458463 & 0{,}46019978 \end{bmatrix}$$

5.3.2 Barycentric Mean of 1950.0 to Barycentric Mean of 2000

The transformation from barycentric B1950.0 to barycentric J2000 includes the following processes:

1. Removal of the terms of elliptic aberration.
2. Rotation to the dynamical equinox of B1950.0
3. Correcting the proper motions for the equinox motion and the change in the value of precession
4. Changing from tropical to Julian centuries for the time scale of proper motions
5. Updating of positions to the epoch of J2000
6. Precession of positions and proper motions from B1950.0 to J2000.

For further details about this transformation, refer to:

- ALMAN05(B32)
- Astronomical and Astrophysical Journal 128, 263-267 (1983)

5.3.3 Barycentric Mean of 2000 to Geocentric Mean of 2000

The transformation from the Barycentric Mean of 2000 to the Geocentric Mean of 2000 reference frame is calculated with the following expressions (Figure 4):

$$\vec{r}_E = \vec{r}_B - \vec{r}_{B,\,Earth}$$

$$\vec{v}_E = \vec{v}_B - \vec{v}_{B,\,Earth}$$

where $\vec{r}_E$ and $\vec{v}_E$ are the position and velocity vectors in the Geocentric Mean of 2000 reference frame, $\vec{r}_B$ and $\vec{v}_B$ are the position and velocity vectors in the Barycentric Mean of 2000 reference frame, and $\vec{r}_{B,\,Earth}$ and $\vec{v}_{B,Earth}$ are the position and velocity vectors of the Earth in the Barycentric Mean of 2000 reference frame.

$\vec{r}_{B,\,Earth}$ and $\vec{v}_{B,Earth}$ are calculated according to BOWRING reference.

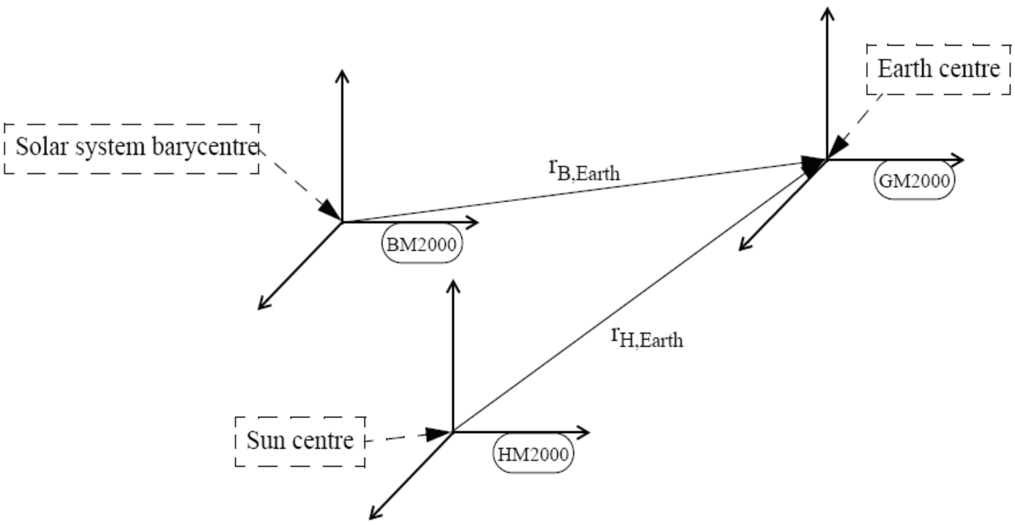


Figure 5: Transformations between BM2000, HM2000 and GM2000 reference frames

5.3.4 Heliocentric Mean of 2000 to Geocentric Mean of 2000

The transformation from the Heliocentric Mean of 2000 to the Geocentric Mean of 2000 reference frame is calculated with the following expressions (Figure 5):

$$\vec{r}_E = \vec{r}_H - \vec{r}_{H,\,Earth}$$
$$\vec{v}_E = \vec{v}_H - \vec{v}_{H,\,Earth}$$

where $\vec{r}_E$ and $\vec{v}_E$ are the position and velocity vectors in the Geocentric Mean of 2000 reference frame, $\vec{r}_H$ and $\vec{v}_H$ are the position and velocity vectors in the Heliocentric Mean of 2000 reference frame, and $\vec{r}_{H,\,Earth}$ and $\vec{v}_{H,Earth}$ are the position and velocity vectors of the Earth in the Heliocentric Mean of 2000 reference frame.

$\vec{r}_{H,\,Earth}$ and $\vec{v}_{H,Earth}$ are calculated according to BOWRING reference.

5.3.5 Geocentric Mean of 2000 to Mean of Date

The transformation from the Geocentric Mean of 2000 to the Mean of Date reference frame is performed with the following expression (Figure 6):

$$\vec{r}_m = R_z\left(-\frac{\pi}{2} - z\right)R_x(\theta)R_z\left(\frac{\pi}{2} - \zeta\right)\vec{r}_{J2000}$$

where $\vec{r}_m$ and $\vec{r}_{J2000}$ are the position vector in the Mean of Date and the Mean of 2000 reference frame, respectively.

The rotation angles of the precession model are calculated as follows (OAD_TIME reference):

$$\xi = 0.6406161T + 0.0000839T^2 + 0.0000050T^3 [deg]$$
$$z = 0.6406161T + 0.0003041T^2 + 0.0000051T^3 [deg]$$
$$\vartheta = 0.5567530T - 0.0001185T^2 - 0.0000116T^3 [deg]$$

where T is the TDB time expressed in the Julian centuries format (1 Julian century = 36525 days).

However, the precession motion is so slow that the UTC time can be used instead of the TDB time, and therefore T can be calculated from t, the UTC time expressed in the MJD2000 format, with the following expression:

T = (t - 0.5)/36525 [Julian centuries]

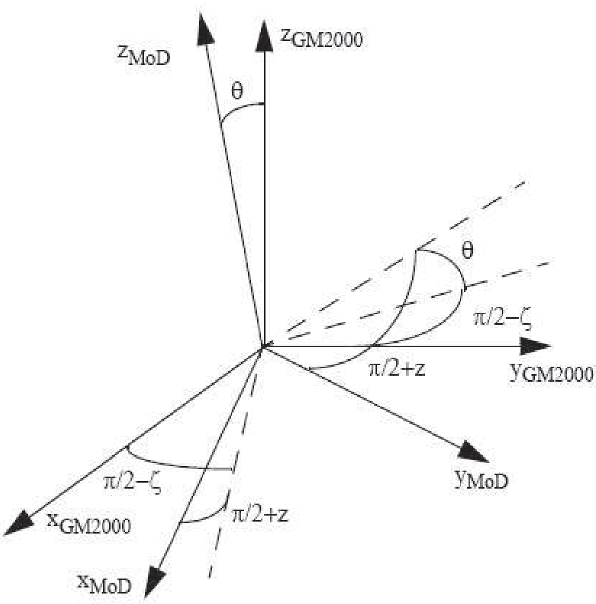


Figure 6: Transformation between GM200 and MoD reference frames

5.3.6 Mean of Date to True of Date

The transformation from the Mean of Date to the True of Date reference frame is performed with the following expression (Figure 7):

$$\bar{r}_t = R_z(-\delta\mu)R_x(-\delta\varepsilon)R_y(\delta\upsilon)\bar{r}_m$$

where $\vec{r}_t$ and $\vec{r}_m$ are, respectively, the position vector in the True of Date and the Mean of Date reference frame.

The rotation angles of the simplified nutation model are calculated with (OAD_TIME reference):

$$\delta\mu = \delta\psi\cos\varepsilon$$

$$\delta\upsilon = \delta\psi\sin\varepsilon$$

where ε is the obliquity of the ecliptic at the epoch J2000:

$\varepsilon = 23.439291$ [deg]

and $\delta\varepsilon$ and $\delta\psi$ is expressed by the *Wahr* model taking only the nine largest terms, and using UT1 instead of TDB as the time reference.

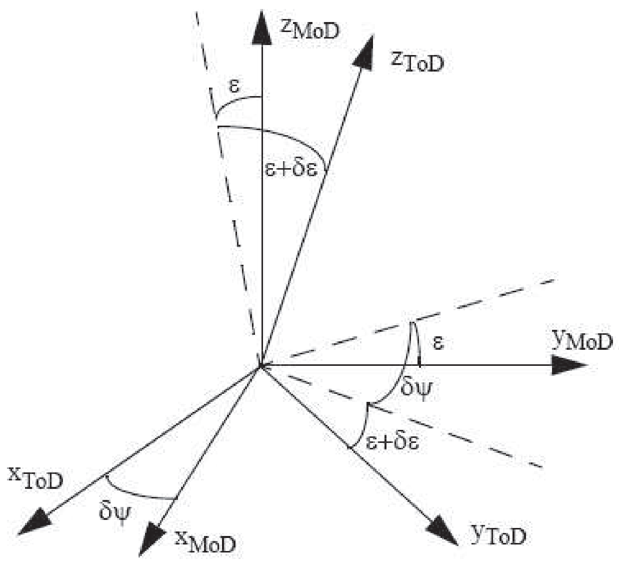


Figure 7: Transformation between MoD and ToD reference frames

5.3.7 True of Date to Pseudo Earth Fixed

The transformation from the True of Date to the Pseudo Earth fixed reference frame is performed with the following expression (Figure 8):

$$\vec{r}_{pe} = R_z(H)\vec{r}_t$$

where $\vec{r}_{pe}$ and $\vec{r}_t$ are, respectively, the position vector in the Pseudo Earth fixed and in the True of Date reference frames.

The ***Earth rotation angle H*** is the sum of the Greenwich sidereal angle and a small term from the nutation in the longitude of the equinox.

The Greenwich sidereal angle moves with the daily rotation of the Earth and is calculated with the Newcomb's formula according to international conventions as a third order polynomial, although the third order term will be neglected in our calculations.

The nutation term is calculated with the simplified nutation model (see section 5.1.7).

$$H = G + \delta\mu$$

$$G = 99.96779469 + 360.9856473662860T + 0.29079\,x\,10^{-12}\,T^2[deg]$$

where T is the UT1 time expressed in the MJD2000 format.

Note that the transformation from the Mean of Date to the Pseudo Earth fixed reference frame can be performed in one step being the δμ rotation term cancelled out:

$$\vec{r}_{pe} = R_z(G)\,R_x(-\delta\varepsilon)\,R_y(\delta\upsilon)\,\vec{r}_{qm}$$

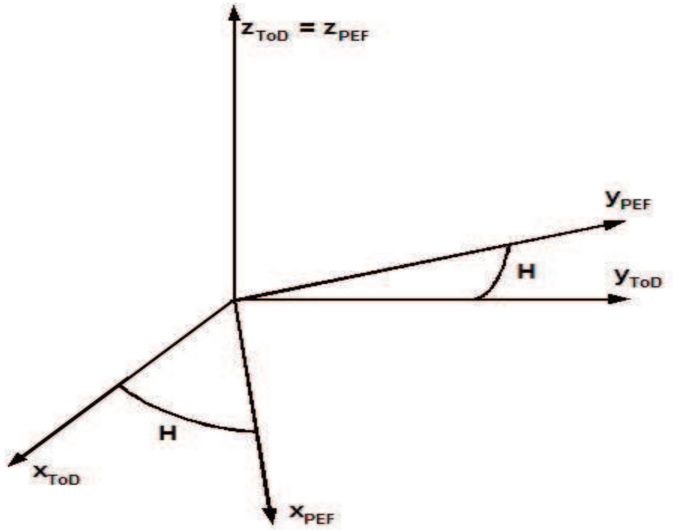


Figure 8: Transformation between ToD and PEF reference frames

5.3.8 Pseudo Earth Fixed to Earth Fixed

The transformation from the Pseudo Earth Fixed to the Earth fixed reference frame is performed with the following expression (Figure 9):

$$\vec{r}_{EF} = R_y(-X)\,R_x(-Y)\,\vec{r}_{PEF}$$

where $\vec{r}_{EF}$ and $\vec{r}_{PEF}$ are, respectively, the position vector in the Earth fixed and in the Pseudo Earth Fixed reference frames; X and Y are the polar motion parameters (measured and predicted by the IERS).

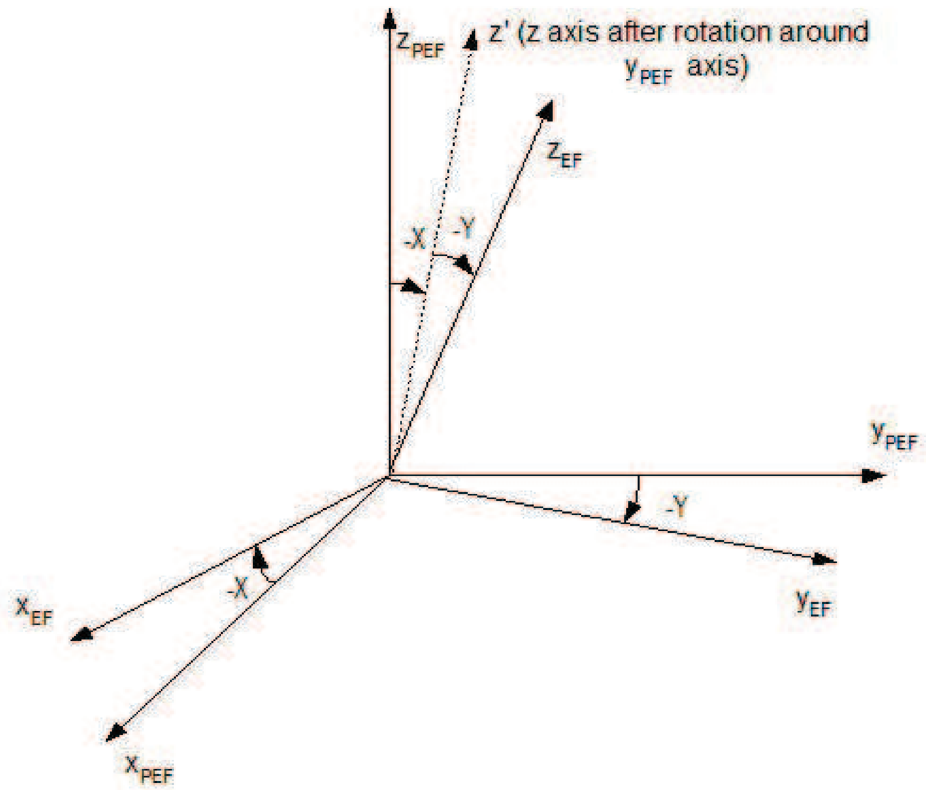


Figure 9: *Transformation between PEF and EF reference frames*

5.4 Satellite Reference Frames Transformations

There is not a general rule for transforming from one satellite reference frame to another. The attitude computation provides the transformation matrix from the satellite frame to an inertial reference frame. The following picture identifies the CFI-specific reference frames transformations that are relevant for the Earth Observation missions:

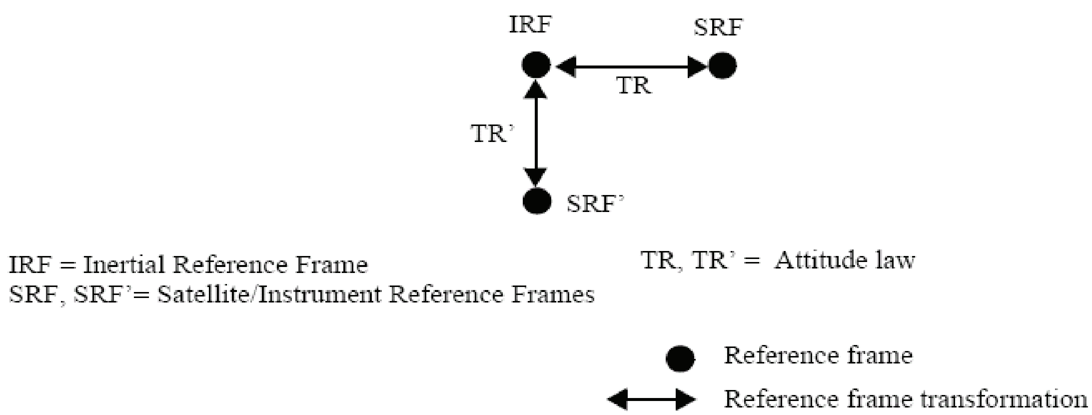


Figure 10: CFI-specific Reference Frames Transformations